

16. [Factors / Multiples / Primes]

Skill 16.1 Recognising a factor.

MM7 1 1 2 2 3 3 4 4
MM8 1 1 2 2 3 3 4 4

To decide if a number is a factor of another number, divide the larger number by the smaller number. If the remainder is 0, then the smaller number is a factor of the larger number.

Q. Is 4 a factor of 622?

A. *No*

Divide 622 by 4 and check the remainder:

$622 \div 4 = 155$, remainder 2, so 4 isn't a factor of 622.

OR

4 is a factor of all numbers where the last two digits are divisible by 4. The last two digits of 622 are 22, which is not divisible by 4. So 4 is not a factor of 622.

Q. List all the factors of 12 in ascending order.

A. *1, 2, 3, 4, 6, 12*

1, 2, 3, 4, 6, 12 are all factors of 12 because they all divide into 12 with no remainder.

Ascending order means from smallest to largest.

Q. The number 8 has exactly four factors: 1, 2, 4 and 8. Find the next number after 8 that has exactly four factors.

A. *10*

Use trial and error.

9 has three factors, 1, 3 and 9.

10 has four factors, 1, 2, 5 and 10, so the answer is 10.

a) Is 5 a factor of 1065?

$1065 \div 5 = 213$

Yes

b) Is 2 a factor of 711?

$711 \div 2 = 355 \text{ r}1$

No

c) Is 3 a factor of 468?

d) Is 6 a factor of 932?

e) Is 4 a factor of 2880?

f) Is 9 a factor of 3054?

g) List all the factors of 21 in descending order.

1, 3, 7, 21

h) List all the factors of 27 in ascending order.

i) List all the factors of 32 in ascending order.

j) The number 15 has exactly four factors: 1, 3, 5 and 15. Find the next number after 15 that has exactly four factors.

Q. List all the common factors of 12 and 18.

A. 1, 2, 3, 6

List all the factors of 12: 1, 2, 3, 4, 6, 12

List all the factors of 18: 1, 2, 3, 6, 9, 18

The common factors of 12 and 18 are 1, 2, 3 and 6.

Q. List all the common factors of 20 and 36.

A. 1, 2, 4

List all the factors of 20: 1, 2, 4, 5, 10, 20

List all the factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

The common factors of 20 and 36 are 1, 2 and 4.

a) List all the common factors of 24 and 32.

Factors of 24

1, 2, 3, 4, 6, 8, 12, 24

Factors of 32

1, 2, 4, 8, 16, 32

Common factors of 24 and 32

1, 2, 4, 8

b) List all the common factors of 30 and 45.

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c) List all the common factors of 18 and 63.

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d) List all the common factors of 40 and 60.

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e) List all the common factors of 12 and 15.

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f) List all the common factors of 18 and 24.

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g) List all the common factors of 12 and 36.

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h) List all the common factors of 12 and 20.

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i) List all the common factors of 45 and 75.

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Q. What is the highest common factor (HCF) of 20 and 24?

A. 4

List all the factors of 20:

1, 2, 4, 5, 10, 20

List all the factors of 24:

1, 2, 3, 4, 6, 8, 12, 24

The common factors of 20 and 24 are 1, 2 and 4, so the highest common factor is 4.

a) What is the highest common factor (HCF) of 12 and 15?

Factors of 12

1, 2, 3, 4, 6, 12

Factors of 15

1, 3, 5, 15

HCF of 12 and 15

3

b) What is the highest common factor (HCF) of 12 and 36?

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c) What is the highest common factor (HCF) of 20 and 30?

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d) What is the highest common factor (HCF) of 21 and 49?

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e) What is the highest common factor (HCF) of 15 and 25?

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f) What is the highest common factor (HCF) of 24 and 30?

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A multiple of a number is the product of that number with another number, e.g. 2 and 4. Because $2 \times 4 = 8$, then 8 is a multiple of 2 and 4.

<p>Q. List all the multiples of 7 up to 50.</p>	<p>A. 7, 14, 21, 28, 35, 42, 49</p>	<p>Start multiplying 7 by 1, 2, 3, and so on. The products are multiples of 7: $7 \times 1 = 7$ $7 \times 2 = 14$ $7 \times 3 = 21$ $7 \times 4 = 28$ and so on till 50.</p>
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<p>Q. List all the common multiples of 4 and 8 up to 60.</p>	<p>A. 8, 16, 24, 32, 40, 48, 56</p>	<p>The multiples of 4 up to 60 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60 The multiples of 8 up to 60 are: 8, 16, 24, 32, 40, 48, 56 The common multiples of 4 and 8 up to 60 are 8, 16, 24, 32, 40, 48, 56.</p>
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a) List all the multiples of 8 up to 50.

8, 16, 24, 32, 40, 48

b) List all the multiples of 4 up to 30.

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c) List all the common multiples of 5 and 7 up to 40.
Multiples of 5 up to 40
5, 10, 15, 20, 25, 30, 35, 40
Multiples of 7 up to 40
7, 14, 21, 28, 35
Common multiples of 5 and 7 up to 40
35

d) List all the common multiples of 4 and 6 up to 40.

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e) List all the common multiples of 2 and 5 up to 30.

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f) List all the common multiples of 6 and 8 up to 50.

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<p>Q. What is the lowest common multiple (LCM) of 6 and 9?</p>	<p>A. 18</p>	<p>List multiples of 6 in ascending order: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60</p> <p>List multiples of 9 in ascending order: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90</p> <p>Common multiples of 6 and 9 are 18, 36 and 54, so the lowest common multiple is 18.</p>
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<p>Q. What is the lowest common multiple (LCM) of 5 and 7?</p>	<p>A. 35</p>	<p>List multiples of 5 in ascending order: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70</p> <p>List multiples of 7 in ascending order: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70</p> <p>Common multiples of 5 and 7 are 35 and 70, so the lowest common multiple is 35.</p> <p>OR</p> <p>In this case the LCM of 5 and 7 is their product: $5 \times 7 = 35$</p>
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a) What is the lowest common multiple (LCM) of 4 and 6?

Multiples of 4
4, 8, 12, 16, 20, 24
.....
Multiples of 6
6, 12, 18, 24, 30, 36
.....
LCM of 4 and 6
12
.....

b) What is the lowest common multiple (LCM) of 6 and 10?

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c) What is the lowest common multiple (LCM) of 8 and 12?

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d) What is the lowest common multiple (LCM) of 12 and 20?

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e) What is the lowest common multiple (LCM) of 4 and 9?

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f) What is the lowest common multiple (LCM) of 10 and 12?

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Any number that has exactly two factors (1 and the number itself) is a prime number.
1 is not a prime number.

Q. List all prime numbers between 6 and 30.	A. 7, 11, 13, 17, 19, 23, 29	<i>The only prime numbers between 6 and 30 are 7, 11, 13, 17, 19, 23 and 29.</i>
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Q. What is the next prime number after 53?	A. 59	<i>54, 55, 56, 57, 58 are not prime numbers, so 59 is the next prime number after 53.</i>
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a) List all prime numbers between 15 and 35.
17, 19, 23, 29, 31

b) List all prime numbers between 20 and 40.
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c) List all prime numbers between 40 and 60.
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d) List all prime numbers between 45 and 65.
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e) List all prime numbers between 70 and 100.
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f) List all prime numbers between 55 and 75.
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g) What is the next prime number after 40?
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h) What is the next prime number after 50?
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i) What is the next prime number after 60?
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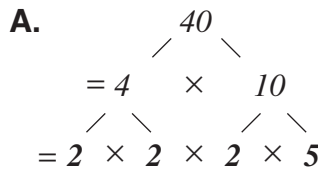
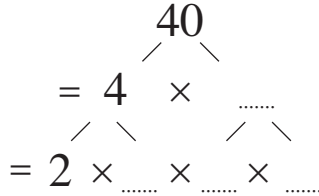
j) What is the next prime number after 70?
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k) What is the next prime number after 80?
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l) What is the next prime number after 90?
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Factor trees are an easy way to write a number as a product of prime numbers.

Q. Show how 40 can be expressed as a product of prime numbers.



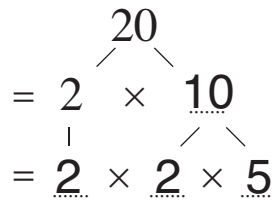
As long as you don't use 1 as a factor, you can use any factors but you will always end up with prime factors at the bottom of the factor tree.

There are two branches that start from the top number of the factor tree. They represent two factors of 40, 4 and 10.

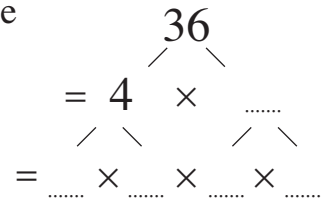
Then 4 and 10 can be expressed as products of two of their factors:
 $2 \times 2 = 4$ and $2 \times 5 = 10$

The tree shows how 40 can be expressed as a product of prime numbers: $2 \times 2 \times 2 \times 5 = 40$

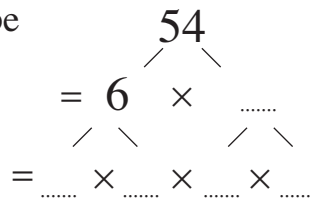
a) Show how 20 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



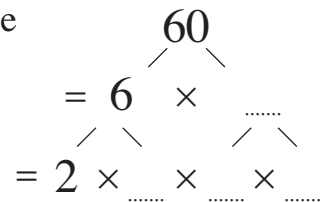
b) Show how 36 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



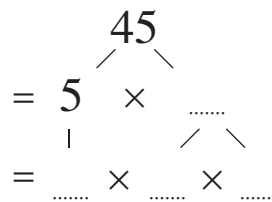
c) Show how 54 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



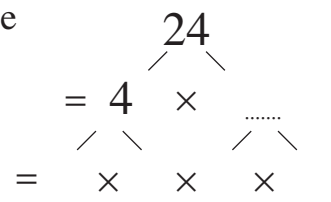
d) Show how 60 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



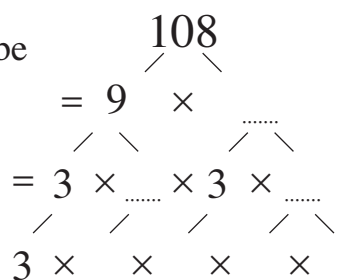
e) Show how 45 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



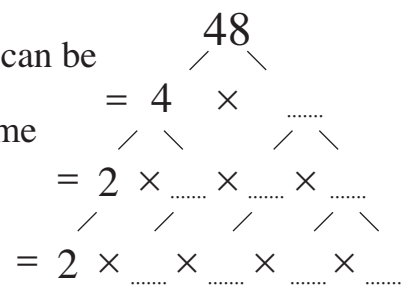
f) Show how 60 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



g) Show how 72 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



h) Show how 48 can be expressed as a product of prime numbers.
 [Complete the factor tree.]



Q. List the prime factors of 60.

A. 2, 3, 5

OR

$$\begin{array}{c}
 60 \\
 \swarrow \quad \searrow \\
 6 \quad \times \quad 10 \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 2 \quad \times \quad 3 \quad \times \quad 2 \quad \times \quad 5
 \end{array}$$

Read the prime factors of 60 from the bottom line of the tree: 2, 3, 5

List the factors of 60:

1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

The only prime factors of 60 are 2, 3, 5.

OR

Use a factor tree.

a) List the prime factors of 75.

3, 5

$$\begin{array}{c}
 75 \\
 \swarrow \quad \searrow \\
 3 \quad \times \quad 15 \\
 | \quad \quad \swarrow \quad \searrow \\
 3 \quad \times \quad 3 \quad \times \quad 5
 \end{array}$$

b) List the prime factors of 90.

$$\begin{array}{c}
 90 \\
 \swarrow \quad \searrow \\
 = 9 \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 = \dots \times \dots \times \dots \times \dots
 \end{array}$$

c) List the prime factors of 140.

$$\begin{array}{c}
 140 \\
 \swarrow \quad \searrow \\
 = 10 \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 = \dots \times \dots \times \dots \times \dots
 \end{array}$$

d) List the prime factors of 125.

$$\begin{array}{c}
 125 \\
 \swarrow \quad \searrow \\
 5 \quad \times \quad \dots \\
 | \quad \quad \swarrow \quad \searrow \\
 5 \quad \times \quad \dots \quad \times \quad \dots
 \end{array}$$

e) List the prime factors of 80.

$$\begin{array}{c}
 80 \\
 \swarrow \quad \searrow \\
 = 8 \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 = 2 \times \dots \times \dots \times \dots \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 = 2 \times \dots \times \dots \times \dots \times \dots
 \end{array}$$

f) List the prime factors of 150.

$$\begin{array}{c}
 150 \\
 \swarrow \quad \searrow \\
 = 10 \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 = \dots \times \dots \times \dots \times \dots
 \end{array}$$

g) List the prime factors of 42.

$$\begin{array}{c}
 42 \\
 \swarrow \quad \searrow \\
 \dots \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad | \\
 \dots \quad \times \quad \dots \quad \times \quad \dots
 \end{array}$$

h) List the prime factors of 110.

$$\begin{array}{c}
 110 \\
 \swarrow \quad \searrow \\
 \dots \quad \times \quad \dots \\
 \swarrow \quad \searrow \quad | \\
 \dots \quad \times \quad \dots \quad \times \quad \dots
 \end{array}$$

i) List the prime factors of 70.

$$\begin{array}{c}
 70 \\
 \swarrow \quad \searrow \\
 \dots \quad \times \quad \dots \\
 | \quad \quad \swarrow \quad \searrow \\
 \dots \quad \times \quad \dots \quad \times \quad \dots
 \end{array}$$

Q. Express 12 as a product of its prime factors.

A. $12 = 2 \times 6$
 $= 2 \times (2 \times 3)$
 $= 2 \times 2 \times 3$

OR
$$\begin{array}{c} 12 \\ / \quad \backslash \\ 2 \quad \times \quad 6 \\ | \quad \quad / \quad \backslash \\ 2 \quad \times \quad 2 \quad \times \quad 3 \end{array}$$

From the tree
 $12 = 2 \times 2 \times 3$

Product means multiplication.
 First write 12 as a product of ANY two of its factors, excluding 1.
 For example $12 = 2 \times 6$. Note that 2 is a prime number but 6 is not.

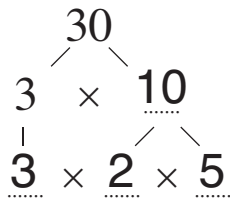
Then write 6 as a product of two of its factors, excluding 1.
 For example $6 = 2 \times 3$
 OR
 Use a factor tree.

a) Express 30 as a product of its prime factors.

$30 = 3 \times 10$

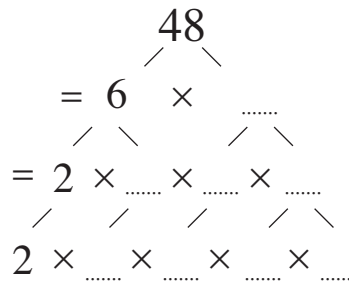
 $= 3 \times (2 \times 5)$

 $= 2 \times 3 \times 5$



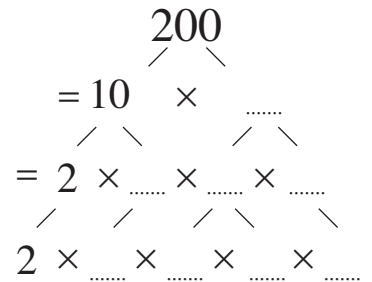
b) Express 48 as a product of its prime factors.

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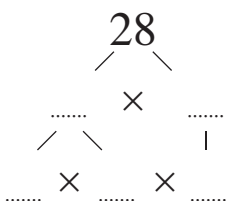
c) Express 200 as a product of its prime factors.

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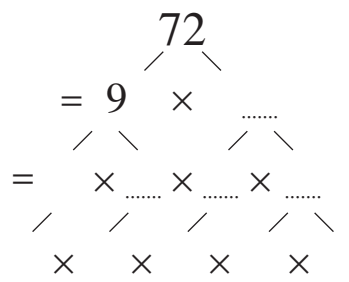
d) Express 28 as a product of its prime factors.

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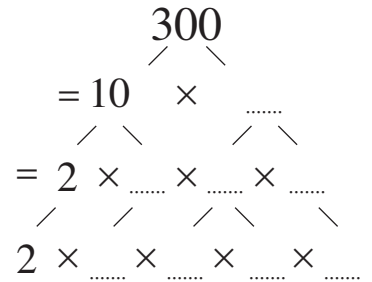
e) Express 72 as a product of its prime factors.

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f) Express 300 as a product of its prime factors.

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Q. Express 120 as a product of its prime factors using exponential notation.

A. $120 = 12 \times 10$
 $= (2 \times 2 \times 3) \times (2 \times 5)$
 $= 2 \times 2 \times 2 \times 3 \times 5$
 $= 2^3 \times 3 \times 5$

Write 120 as a product of ANY two of its factors, excluding 1.
 For example $120 = 12 \times 10$

Then write both 12 and 10 as products of their prime factors.

$12 = 2 \times 2 \times 3$

$10 = 2 \times 5$

Group like numbers in ascending order.
 Use exponential notation: $2 \times 2 \times 2 = 2^3$

a) Express 8 as a product of its prime factors using exponential notation.

$8 = 2 \times 4$

 $= 2 \times 2 \times 2$

 $= 2^3$

b) Express 36 as a product of its prime factors using exponential notation.

$36 = 4 \times 9$

 $= (2 \times 2) \times (3 \times 3)$

 $= 2^2 \times 3^2$

c) Express 45 as a product of its prime factors using exponential notation.

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d) Express 18 as a product of its prime factors using exponential notation.

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e) Express 63 as a product of its prime factors using exponential notation.

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f) Express 72 as a product of its prime factors using exponential notation.

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g) Express 90 as a product of its prime factors using exponential notation.

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h) Express 100 as a product of its prime factors using exponential notation.

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i) Express 220 as a product of its prime factors using exponential notation.

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