13. [Exploring Number]

Skill 13.1 Using "order of operations" involving a mix of ( ), ×, ÷, + or –

MM9 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Simplify within the brackets.
- Multiply (×) and/or divide (÷) in order from left to right.
- Add (+) and/or subtract (–) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$ *simplify the brackets first*
$= 18 \div 6 + 2$ *division before addition*
$= 3 + 2$
$= \mathbf{5}$

a) $6 + 12 \div 4 \times 3 =$

$= 6 + 3 \times 3$

$= 6 + 9$ $=$ ☐

b) $6 \times 15 - 8 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

c) $5 + 12 \div 6 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

d) $3 \times (5 - 3) \times 8 =$

$=$ .....

$=$ ..... $=$ ☐

e) $(15 + 8) - (7 + 6) =$

$=$ .....

$=$ ..... $=$ ☐

f) $120 \div 5 - 6 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

g) $22 - 8 - (11 - 4) =$

$=$ .....

$=$ ..... $=$ ☐

h) $20 - (15 - 9) + 6 =$

$=$ .....

$=$ ..... $=$ ☐

i) $6 \times (14 + 7) =$

$=$ .....

$=$ ..... $=$ ☐

j) $8 \times 5 \div (7 - 3) =$

$=$ .....

$=$ ..... $=$ ☐

k) $4 + (9 - 4) \times 7 =$

$=$ .....

$=$ ..... $=$ ☐

l) $36 - 2 \times (12 + 5) =$

$=$ .....

$=$ ..... $=$ ☐

Skill 13.2 Using "order of operations" involving powers and ( ), ×, ÷, + or –

- Simplify within the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (–) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$
multiply within brackets first
$= (6 + 10)^2$
add with the brackets
$= 16^2$
$= \mathbf{256}$

a) $(3 + 5)^2 =$

$= 8^2$ ..... $=$ ☐

b) $(12 - 7)^2 =$

$=$ ..... $=$ ☐

c) $(5 + 5 \times 3)^2 =$

$=$ .....

$=$ ..... $=$ ☐

d) $(2 \times 4 + 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

e) $(2 + 8)^2 \div 4 =$

$=$ .....

$=$ ..... $=$ ☐

f) $(7 + 5)^2 \div 8 =$

$=$ .....

$=$ ..... $=$ ☐

g) $5 + (12 - 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

h) $8 + (13 - 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

i) $(4 \times 2 + 2)^2 =$

$=$ .....

$=$ ..... $=$ ☐

j) $(3 \times 4 + 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

k) $3 + (1 + 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

l) $6 + (7 + 1)^2 =$

$=$ .....

$=$ ..... $=$ ☐

Skill 13.3 Completing number patterns.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
- Define the rule of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Hints: Every number pattern is created by a rule involving numbers and operations. Counting numbers, even numbers and odd numbers have patterns themselves that can become part of the rule (see below).

Q. Complete the pattern:

3 , 5 , 9 , 15 , 23 , ____ , ____

A. *3 , 5 , 9 , 15 , 23 ,*

+2 +4 +6 +8

Rule: Add 2, then 4, then 6, then 8, etc.
The pattern is formed by adding consecutive even numbers.

23 + 10 = 33

33 + 12 = 45

3 , 5 , 9 , 15 , 23 , ***33 , 45***

a) Complete the pattern:

1 , 3 , 9 , 27 , 81 , ____ , ____

×3 ×3 ×3 ×3

81 × 3 = 243 *243 × 3 = 729*

b) Complete the pattern:

3 , 6 , 12 , 24 , 48 , ____ , ____

×2 ×2 ×2 ×2

48 × 2 =

c) Complete the pattern:

$\frac{1}{36}$, $\frac{1}{6}$, 1 , 6 , ____ , ____

×6 ×6 ×6

d) Complete the pattern:

$\frac{1}{25}$, $\frac{1}{5}$, 1 , 5 , ____ , ____

e) Complete the pattern:

11 , 13 , 16 , 20 , ____ , ____

+2 +3

f) Complete the pattern:

2 , 3 , 5 , 8 , 12 , ____ , ____

g) Complete the pattern:

3 , 5 , 9 , 15 , ____ , ____

h) Complete the pattern:

1 , 3 , 7 , 13 , ____ , ____

i) Complete the pattern:

1 , 4 , 10 , 19 , ____ , ____

j) Complete the pattern:

5 , 6 , 9 , 14 , 21 , ____ , ____

Skill 13.4 Finding a term in a number pattern.

- Draw a table and list the given terms and also the position each term occupies in the pattern.
- Look for a relationship between consecutive terms and/or between the term and its position in the pattern.
- Based on this relationship, find the requested term in the pattern.

Q. Find the 30th term in the pattern
2 , 4 , 6 , 8 , 10 , ...

A.

position	1st	2nd	3rd	4th	5th	.....	30th
term	2	4	6	8	10		?
relationship	1 × 2	2 × 2	3 × 2	4 × 2	5 × 2		30 × 2

×2

Relationship: each term = twice its position.
The 30th term of the pattern is 2 × 30 = ***60***

a) Find the 20th term in the pattern
5 , 7 , 9 , 11 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	5	7	9	11	13		?
relationship	1×2+3	2×2+3	3×2+3	4×2+3	5×2+3		20×2+3

×2+3

Relationship: term = position × 2 + 3

20th term = 20 × 2 + 3 = ☐

b) Find the 15th term in the pattern
5 , 10 , 15 , 20 , 25 , ...

position	1st	2nd	3rd	4th	5th	.....	15th
term	5	10	15	20	25		?
relationship	1 × 5	2 × 5	3 × 5	4 × 5	5 × 5		

?

Relationship: term = ...

15th term = ... = ☐

c) Find the 20th term in the pattern
8 , 13 , 18 , 23 , 28 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	8	13	18	23	28		?
relationship	1×5+3	2×5+3	3×5+3	4×5+3	5×5+3		

?

Relationship: term = ...

20th term = ... = ☐

d) Find the 25th term in the pattern
4 , 6 , 8 , 10 , 12 , ...

position	1st	2nd	3rd	4th	5th	.....	25th
term	4	6	8	10	12		?
relationship							

?

Relationship: term = ...

25th term = ... = ☐

e) Find the 20th term in the pattern
1 , 4 , 7 , 10 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	1	4	7	10	13		?
relationship							

?

Relationship: term = ...

20th term = ... = ☐

f) Find the 8th term in the pattern
1 , 2 , 4 , 8 , 16 , ...

position	1st	2nd	3rd	4th	5th	.....	8th
term	1	2	4	8	16		?
relationship							

?

Relationship: term = ...

8th term = ... = ☐

Skill 13.5 Rounding decimal numbers to a given place.

To round off terminating decimals to a given place:

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

To round off recurring decimals to a given place:

- Write the first 4 digits after the decimal point. (see skill 7.6, page 72)
- Apply the procedure described above for terminating decimals.

Q. Round off $0.4\overline{6}$ to the nearest thousandth.

A. $0.4\overline{6} = 0.4666...$ (6 is repeating indefinitely)

$0.4666..$ (circle the fourth digit)

$\approx$ ***0.467*** (6 ≥ 5, round up by adding 1 to 6)

a) Round off 0.13 to the nearest tenth.

0.13 (3 < 5, round down by keeping 1) 0.1

b) Round off 7.89 to the nearest tenth.

.....

c) Round off 12.45 to the nearest tenth.

.....

d) Round off 31.5841 to the nearest hundredth.

31.5841 (4 < 5, round down by keeping 8) 31.58

e) Round off 24.793 to the nearest hundredth.

.....

f) Round off 4.231 to the nearest hundredth.

.....

g) Round off 3.859 to the nearest tenth.

.....

h) Round off 50.296 to the nearest hundredth.

.....

i) Round off $4.\overline{7}$ to the nearest hundredth.

$4.\overline{7} = 4.7777...$ ($7 \geq 5$, round up by adding 1 to 7) $\approx$ 4.78

j) Round off $3.\overline{42}$ to the nearest hundredth.

$\approx$

k) Round off $0.\overline{6}$ to the nearest hundredth.

$\approx$

l) Round off $1.7\overline{3}$ to the nearest thousandth.

$\approx$

m) Round off $4.2\overline{8}$ to the nearest thousandth.

$\approx$

n) Round off $0.\overline{16}$ to the nearest thousandth.

$\approx$

Skill 13.6 Writing rational approximations of simple irrational numbers.

irrational number $\sqrt{2} \approx 1.41421356$ rational approximation

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the the nearest hundredth.

Q. cos 45° ≈ 0.70711
Write the rational approximation of cos 45° correct to two decimal places.

A. *0.707*11
circle the third digit
7 ≥ 5 round up by adding 1 to 0
0.71

a) $\sqrt{12} \approx 3.46410162$
Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

*3.464*10162 *4 < 5 round down by keeping 6* ≈ 3.46

b) $\sqrt{20} \approx 4.47213595$
Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

..... ≈ .....

c) $\sqrt{24} \approx 4.89897949$
Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

..... ≈ .....

d) $\sqrt{30} \approx 5.47722558$
Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

..... ≈ .....

e) $\pi \approx 3.14159265$
Write the rational approximation of π correct to three decimal places.

..... ≈ .....

f) $\varphi \approx 1.61803398$
Write the rational approximation of φ correct to three decimal places.

..... ≈ .....

g) sine 15° ≈ 0.25882
Write the rational approximation of sine 15° correct to three decimal places.

..... ≈ .....

h) tan 60° ≈ 1.73205
Write the rational approximation of tan 60° correct to three decimal places.

..... ≈ .....

i) e ≈ 2.71828182 (Euler's number)
Write the rational approximation of e correct to two decimal places.

..... ≈ .....

j) $\sqrt{10} \approx 3.16227766$
Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

..... ≈ .....

Skill 13.7 Writing very large and very small numbers in standard form (scientific notation).

243 000	=	2.43×10^5	0.000802	=	8.02×10^{-4}
Basic Number		**Standard Form**	**Basic Number**		**Standard Form**
Very large		Product of: Number ≥ 1 and < 10 Power of 10 with positive exponent	Very small		Product of: Number ≥ 1 and < 10 Power of 10 with negative exponent

To find the number greater than or equal to 1 and less than 10:
- Move the decimal point after the first non-zero digit and count the places moved.
- Write this number ignoring the zeros at either end.

To find the power of 10:
- Write base 10 with an exponent equal to the number of places moved.
- Check the sign of the exponent: Decimal point moves left ⇒ positive exponent.
  Decimal point moves right ⇒ negative exponent

Q. Write 384 000 000 m, the distance from earth to moon, in scientific notation.

A. $384\,000\,000$ (8 places)
$= 3.84 \times 100\,000\,000$ (8 zeros)
$= \mathbf{3.84 \times 10^8}$ (number ≥ 1 and < 10; exponent = +8 because of the 8 places and large original number (> 3.84))

a) 73 000 written in standard form is:
A) 7.3×10^{-4} B) 0.73×10^5 C) 7.3×10^4

$73\,000$ (4 places; exponent = +4)
$= 7.3 \times 10\,000 = 7.3 \times 10^4$ (4 zeros) = C

b) 5 010 000 written in standard form is:
A) 5.01×10^6 B) 5.1×10^5 C) 5.01×10^5

$5\,010\,000$
= ..... =

c) Some people have as many as 150 000 hair follicles on their head. Write this in scientific notation.

..... =

d) By 2010 the world's population is estimated to reach 6.84 billion people. Write this in standard form.

..... =

e) 0.00304 written in standard form is:
A) 3.4×10^{-4} B) 3.04×10^{-3} C) 3.04×10^{-4}

0.00304 (3 places; exponent = −3)
$= 3.04 \times 1000 = 3.04 \times 10^{-3}$ (3 zeros) =

f) 0.000051 written in standard form is:
A) 5.1×10^{-5} B) 5.1×10^5 C) 5.1×10^{-4}

= ..... =

g) Write 0.00000024 m, the diameter of a human hair strand, in scientific notation.

= ..... =

h) In diamonds the distance between two Carbon atoms is 0.000000142 m. Write this in scientific notation.

= ..... =

Skill 13.8 Writing a number in standard form (scientific notation) as a basic numeral.

2.43×10^5	=	243 000
Standard Form		**Basic Number**
Product of: Number ≥ 1 and < 10 Power of 10 with positive exponent		Very large

8.02×10^{-4}	=	0.000802
Standard Form		**Basic Number**
Product of: Number ≥ 1 and < 10 Power of 10 with negative exponent		Very small

If the power of 10 is **positive:**
- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary. Example: 3.1 = 3.1000
  Hint: By convention 37 = 37. = 37.0

If the power of 10 is **negative:**
- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary. Example: 4.5 = 00004.5
- If the result is less than 1, write a zero in the units place.
  Hint: By convention 0.37 not .37

Q. Write 3.5×10^{-4} m, the diameter of optical fibre, as a basic numeral.

A. 3.5×10^{-4} = (exponent = −4)
= 00003.5×10^{-4} (move decimal point 4 places left)
= **0.00035** (add zeros as place holders)

a) 6.2×10^5 is the scientific notation for:
A) 6200 B) 620 000 C) 6.20000

6.2×10^5 = (exponent = +5)
= 620 000. (5 places right) B

b) 4.12×10^6 is the scientific notation for:
A) 4 120 000 B) 412 000 C) 4.120000

= .....

c) Earth's atmosphere extends upward for 9.65×10^5 m. Write this as a basic numeral.

.....

d) Write 1.3×10^9, China's population in 2006, as a basic numeral.

.....

e) The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
A) 0.0008 B) 8000 C) 0.008

= .....

f) The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
A) 0.00025 B) 0.000025 C) 250 000

= .....

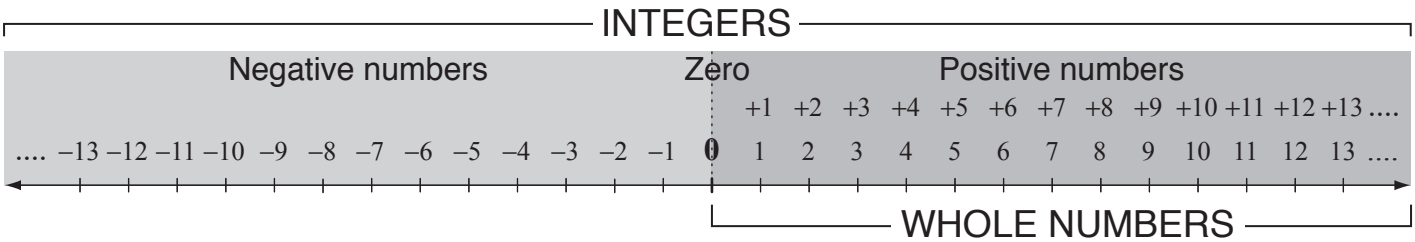
g) Write 2.5×10^{-11} m, the radius of a hydrogen atom, as a basic numeral.

.....

h) Write 5×10^{-7} m, the size of a speck of dust, as a basic numeral.

.....

Skill 13.9 Recognising whole numbers and integers.



- Decide if a number is a whole number or an integer, based on their definition (see Glossary) and hints below.

Hints: *Negative numbers, fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are not whole numbers.*
Any fraction whose numerator is divisible by the denominator is a whole number: $\frac{12}{4} = 3$
Any decimal with only zeros after the decimal point is a whole number: $8.00 = 8$
Fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are not integers.
Any fraction whose numerator is divisible by the denominator is an integer: $\frac{5}{1} = 5$, $\frac{12}{4} = 3$
Any decimal with only zeros after the decimal point is an integer: $8.00 = 8$
Any square root of a perfect square is an integer: $\sqrt{16} = 4$

Q. Choose the whole numbers from this list:
$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

A. *-7 is negative, so not a whole number*
$\frac{8}{2} = 8 \div 2 = 4$ is a whole number
$-\frac{1}{3}$ is a fraction, so not a whole number
-3.6 is a decimal, so not a whole number
So ***$\frac{8}{2}$, 0, 50*** *are whole numbers.*

a) Choose the whole numbers from this list:
$7.43, (89), -5, 3\frac{1}{5}, (14), 0.6$

b) Choose the whole numbers from this list:
$567, 0.73, -4, \frac{3}{10}, 12, 0$

c) Choose the whole numbers from this list:
$1.4142, 18, -5.\overline{9}, \frac{4}{11}, -5, 143$

d) Choose the whole numbers from this list:
$-25, 0.6666..., 34, \frac{5}{7}, -1, 8.93567$

e) Choose the integers from this list:
$-3.5, 11, 2.\overline{14}, -1, 3\frac{2}{7}, 2$

f) Choose the integers from this list:
$3.14, \frac{16}{4}, -3, -0.\overline{72}, \sqrt{25}$

g) Choose the integers from this list:
$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

h) Choose the integers from this list:
$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

Skill 13.10 Recognising rational and irrational numbers.

A number is **rational** if:

- It can be written as a fraction (ratio) of two integers.

Hint: All integers are rational numbers.

$-2,\ 700,\ \sqrt{16},\ \frac{5}{1},\ \frac{25}{5}$

All terminating decimals are rational numbers.

2.16, –5.753469

All recurring decimals are rational numbers.

$0.57575757... = 0.\overline{57}$

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-recurring digits after the decimal point.

Example: 2.52849302953...

Hint: Square roots of prime numbers and rational numbers that are not perfect squares are irrational numbers.

$\sqrt{5},\ \sqrt{18}$

Q. Which numbers are rational?

A) $-\sqrt{\frac{3}{5}}$

B) 0.999...

C) 0.12357102...

D) $\frac{11}{2}$

A. $-\sqrt{\frac{3}{5}}$ *is irrational, because* $\frac{3}{5}$ *is not a perfect square.*

0.999... is rational, because it is a recurring decimal.

0.12357102... is irrational, because it has infinite non-recurring digits after the decimal point.

$\frac{11}{2}$ *is rational, because it is a fraction.*

So ***B and D*** *are rational.*

a) Is $\sqrt{7}$ a rational or an irrational number?

irrational

b) Is $4.12\overline{63}$ a rational or an irrational number?

c) Is $\frac{48}{25}$ a rational or irrational number?

d) Is 1.72430982... a rational or irrational number?

e) Is –60 a whole number, an integer or an irrational number?

f) Is 2.676767... a whole number, an integer or a rational number?

g) Is $-\frac{7}{2}$ a whole number, an integer or a rational number?

h) Is –12 000 a whole number, an integer or an irrational number?

i) Which is an irrational number?

A) 3 B) –2.5

C) $\sqrt{2}$ D) $-\sqrt{4}$

j) Which is an irrational number?

A) $2.\overline{6}$ B) 6.15

C) $\sqrt{7}$ D) $5\frac{3}{10}$

k) Which numbers are rational?

A) $\sqrt{8}$ B) $6.5\overline{9}$

C) –4.131133111333... D) 3.161616...

l) Which numbers are **not** rational?

A) –0.315315315... B) $\sqrt{3}$

C) 2.135791113... D) $\frac{11}{49}$

$\mathbb{R}$ REAL NUMBERS

IRRATIONAL
π, φ, e, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,
2.6293045632....
cos 30°

$\mathbb{Q}$
RATIONAL
$-2\frac{3}{7}$, 3.010101..., $\frac{4}{10}$, 0.56, $\sqrt{\frac{4}{9}}$

$\mathbb{Z}$
Integers
..., –3, –2, –1, 0, 1, 2, 3, ...

$\mathbb{N}$
Natural (Whole Numbers)
0, 1, 2, 3, 4, 5, 6, ...

Hint: Rational numbers include integers, terminating decimals and recurring decimals.
Irrational numbers include infinite non-recurring decimals.

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
Irrational numbers $\subset \mathbb{R}$

Q. Which classes of numbers describe $-\sqrt{81}$?
A) integer and irrational
B) rational and real
C) irrational and rational
D) real and natural

A. $-\sqrt{81} = -9$

integer ✔
rational ✔
real number ✔
natural ✘
irrational ✘

*So **B** is the correct description.*

a) Use true and false to complete this table:

	integer	rational	irrational	real
4.327	false	true	false	true

b) Use true and false to complete this table:

	integer	rational	irrational	real
–500				

c) Use true and false to complete this table:

	integer	rational	irrational	real
π				

d) Use true and false to complete this table:

	integer	rational	irrational	real
$\frac{3}{14}$				

e) Which classes of numbers describe 0.65291...?
A) integer and rational
B) rational and real
C) integer and irrational
D) irrational and real

f) Which classes of numbers describe $-\sqrt{49}$?
A) integer and rational
B) irrational and real
C) integer and irrational
D) rational and irrational

g) Which classes of numbers describe $0.\overline{1}53846\overline{}$?
A) integer and irrational
B) irrational and real
C) integer and rational
D) rational and real

h) Which classes of numbers describe $\frac{257}{43}$?
A) integer and rational
B) irrational and real
C) rational and real
D) rational and irrational