

19. [Factorisation]

Skill 19.1 Factorising by finding the HCF of the coefficients.

MM9 11 22 33 44
MM10 1 22 33 44

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the HCF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise $15a - 24$

A. $15a - 24$

$$3 \times 5 = 15 \text{ and } 3 \times 8 = 24$$

HCF of 15 and 24 is 3

Write the HCF before the ()

Remaining factors are (5a) and (8)

$$= 3(5a - 8) \quad \text{Keep the sign}$$

a) Factorise $4k - 16 = \boxed{4(k - 4)}$

$$4 \times k = 4k \text{ and } 4 \times 4 = 16 \Rightarrow \text{HCF is } 4$$

b) Factorise $4x + 8 = \boxed{}$

c) Factorise $6s + 18 = \boxed{}$

d) Factorise $3u - 15 = \boxed{}$

e) Factorise $9m - 2 = \boxed{}$

f) Factorise $14n + 21 = \boxed{}$

g) Factorise $2y + 10z = \boxed{}$

h) Factorise $4a - 12b = \boxed{}$

i) Factorise $6d + 14e = \boxed{}$

j) Factorise $16uv - 4 = \boxed{}$

k) Factorise $12k - 8l = \boxed{}$

l) Factorise $4g + 4h - 6 = \boxed{}$

m) Factorise $3m - 6n + 9 = \boxed{}$

n) Factorise $10v - 5w + 15 = \boxed{}$

o) Factorise $5h^2 - 10i + 25j = \boxed{}$

p) Factorise $6r^2 - 27s + 9t = \boxed{}$

Skill 19.2 Factorising by finding the HCF of coefficients and pronumerals.

MM9 11 2 2 3 3 4 4
MM10 11 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $18kl - 24k$

A. $18kl - 24k$

$$6 \times 3 = 18 \text{ and } 6 \times 4 = 24$$

HCF of 18 and 24 is 6

k is common to both terms CF is k

Remaining factors are 3 and 4

Write all CF's before the ()

$$= 6k(3l - 4)$$

Keep the sign

a) Factorise $ab + 5b$

$$ab = ba$$

$$b \text{ is common to both terms } = b(a + 5)$$

b) Factorise $de + d$

$$= \boxed{}$$

c) Factorise $7e + ef$

$$= \boxed{}$$

d) Factorise $3st + 4s$

$$= \boxed{}$$

e) Factorise $8ab - 4b$

$$= \boxed{}$$

f) Factorise $15g + 20gh$

$$= \boxed{}$$

g) Factorise $wx - xy$

$$= \boxed{}$$

h) Factorise $2jk + 2kl$

$$= \boxed{}$$

i) Factorise $uv - 3vw$

$$= \boxed{}$$

j) Factorise $8ab + 4bc$

$$= \boxed{}$$

k) Factorise $12qr + 8rs$

$$= \boxed{}$$

l) Factorise $15de - 6ef$

$$= \boxed{}$$

m) Factorise $10cd - 8d$

$$= \boxed{}$$

n) Factorise $15m - 10mn$

$$= \boxed{}$$

o) Factorise $21qr + 14pq$

$$= \boxed{}$$

p) Factorise $6tu + 18uv$

$$= \boxed{}$$

q) Factorise $6xy + 9yz$

$$= \boxed{}$$

r) Factorise $10gh - 25gi$

$$= \boxed{}$$

Skill 19.3 Factorising to simplify expressions involving large numbers.

MM9 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Find the number repeating in both products.
- Write this number in front of the brackets.
Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise and evaluate

$$45 \times 7 + 45 \times 3$$

A. $45 \times 7 + 45 \times 3$ — 45 is repeating

$$= 45 \times (7 + 3)$$

$$= 45 \times 10$$

$$= 450$$

a) Factorise and evaluate

$$99 \times 99 - 98 \times 99$$

$$= 99 \times (99 - 98)$$

$$= 99 \times 1$$

$$= \boxed{99}$$

b) Factorise and evaluate

$$15 \times 14 + 15 \times 6$$

$$= 15 \times (14 + 6)$$

$$=$$

$$=$$

$$\boxed{}$$

c) Factorise and evaluate

$$987 \times 2 + 987 \times 8$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

d) Factorise and evaluate

$$40 \times 8 + 40 \times 12$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

e) Factorise and evaluate

$$23 \times 37 + 23 \times 63$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

f) Factorise and evaluate

$$25 \times 26 + 25 \times 24$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

g) Factorise and evaluate

$$999 \times 9 - 999 \times 8$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

h) Factorise and evaluate

$$87 \times 19 - 87 \times 9$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

i) Factorise and evaluate

$$-4 \times 14 - 4 \times 6$$

$$= -4 \times (14 + 6)$$

$$=$$

$$=$$

$$\boxed{}$$

Both terms are negative
so CF is negative

- + = -

j) Factorise and evaluate

$$-9 \times 33 - 9 \times 67$$

$$=$$

$$=$$

$$=$$

$$\boxed{}$$

Skill 19.4 Factorising involving squared terms.

MM9 11 2 2 3 3 4 4
MM10 11 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $2wx - 12w^2x$

A. $2wx - 12w^2x$

$2 \times 1 = 2$ and $2 \times 6 = 12$

HCF of 2 and 12 is 2

wx is common to both terms CF is wx

Remaining factors are 1 and 6w

Write all CF's before the ()

$= 2wx(1 - 6w)$

Keep the sign

a) Factorise $2j^2k + 5j$

$CF = j$ $= j(2jk + 5)$

Write all CF's outside the ()

b) Factorise $e^2 + 7e$

$=$

c) Factorise $h + 4h^2$

$=$

d) Factorise $m^2 - 9m$

$=$

e) Factorise $3c - 12c^2$

$=$

f) Factorise $4f^2 + 6f$

$=$

g) Factorise $fg^2 + f$

$=$

h) Factorise $10b - 16ab^2$

$=$

i) Factorise $p^2q - 3p$

$=$

j) Factorise $12i - 18hi^2$

$=$

k) Factorise $14bc + 2b^2c$

$=$

l) Factorise $5r^2s - r^2t$

$=$

m) Factorise $vw + 7v^2 - 3vwx$

$=$

n) Factorise $8j^2 - 24jk + 12jl$

$=$

o) Factorise $f^3g^2 + fg^2$

$=$

p) Factorise $p^3q^2 + p^2q + pq$

$=$

Skill 19.5 Factorising negative terms.

MM9 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $-10r^2 - 5r$

A. $-10r^2 - 5r$

$5 \times 2 = 10$ and $5 \times 1 = 5$

HCF of 10 and 5 is 5

r is common to both terms CF is r

$-$ is common to both terms CF is $-$

Remaining factors are r and 1

Write all CF before the ()

$= -5r(r + 1)$

"-" is common to both terms

a) Factorise $-7a - 21 = \boxed{-7(a + 3)}$

$7 \times 1 = 7$ and $7 \times 3 = 21 \Rightarrow$ HCF is 7

b) Factorise $-4k - 12 = \boxed{}$

$4 \times 1 = 4$ and $4 \times 3 = 12 \Rightarrow$ HCF is

c) Factorise $-6g - 15 = \boxed{}$

d) Factorise $-6e - 14 = \boxed{}$

e) Factorise $-2h^2 - 6h = \boxed{}$

f) Factorise $-8z^2 - 28z = \boxed{}$

g) Factorise $-12i^3 - 9ij = \boxed{}$

h) Factorise $-t^3 - 5t^2u = \boxed{}$

i) Factorise $-6bc^2 + 3c^2 = \boxed{}$

j) Factorise $-5x^2y^2 - xy^3 = \boxed{}$

k) Factorise $-2x^3 - 4xy = \boxed{}$

l) Factorise $-4m^3 - 12mn^2 + 18m = \boxed{}$

m) Factorise $-2k^3 + 6k^3l + 8k = \boxed{}$

n) Factorise $-2hi^3 + 3h^2i - 5h^2 = \boxed{}$

Skill 19.6 Factorising by finding binomial factors.

MM9 11 22 33 44
MM10 11 22 33 44

- Find any common factors (CF).

Hint: It might help to think of common factors that are expressions like $(d + 2)$ as a blob ■.

- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $2(r - 1) - r(r - 1)$

A. $2(r - 1) - r(r - 1)$ Consider $(r - 1) = \blacksquare$
 $= 2 \blacksquare - r \blacksquare$ Keep the sign
 $= \blacksquare (2 - r)$
 $= (r - 1)(2 - r)$

- a)** Factorise

$d(d + 2) + 8(d + 2)$ Consider $(d + 2) = \blacksquare$

$= d \blacksquare + 8 \blacksquare$ Keep the sign

$= \blacksquare (d + 8)$ = $(d + 2)(d + 8)$

- b)** Factorise

$2(h - 3) + h(h - 3)$

$= 2 \blacksquare + h \blacksquare$

$= \blacksquare$ =

- c)** Factorise

$5(x + 4) + x(x + 4)$

$= \blacksquare$
 $= \blacksquare$ =

- d)** Factorise

$b(b - 7) + 6(b - 7)$

$= \blacksquare$
 $= \blacksquare$ =

- e)** Factorise

$a(a + 2) - 9(a + 2)$

$= \blacksquare$
 $= \blacksquare$ =

- f)** Factorise

$z(z - 5) - (z - 5)$

$= \blacksquare$
 $= \blacksquare$ =

- g)** Factorise

$j(j + 4) + j + 4$

$= \blacksquare$
 $= \blacksquare$ =

- h)** Factorise

$m(n - 2) + 4(n - 2)$

$= \blacksquare$
 $= \blacksquare$ =

- i)** Factorise

$3x(2x - 5) - 4(2x - 5)$

$= \blacksquare$
 $= \blacksquare$ =

- j)** Factorise

$d(c + 5) - (c + 5)$

$= \blacksquare$
 $= \blacksquare$ =

- k)** Factorise

$q(s - 3) + t(s - 3)$

$= \blacksquare$
 $= \blacksquare$ =

- l)** Factorise

$6v(2w - 1) + 4(2w - 1)$

$= \blacksquare$
 $= \blacksquare$ =

- Begin factorising by grouping the 4 terms in 2 groups of 2.
 - Take out the CF from the first group of 2 and write it in front of the brackets.
 - Take out the CF from the second group of 2 and write it in front of the brackets.
 - Keep the signs.
 - Factorise again by finding the common binomial factor. (see skill 19.6, page 194)
- Hint: It might help to think of binomial factors that are expressions like $(d + 2)$ as a blob ■.*
- Take out the binomial factor or blob and write it in front of the brackets.
 - Write the remaining factors inside the brackets.
 - Check the result by expanding the brackets.

Q. Factorise $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$ *Keep the sign*
 $= m(m + 3) + 5(m + 3)$
 $= m(\blacksquare) + 5(\blacksquare)$ *Consider $(m + 3) = \blacksquare$*
 $= \blacksquare(m + 5)$
 $= (m + 3)(m + 5)$

a) Factorise $c^2 + 8c + 3c + 24$
*Group 2 and 2
Factorise each group*
 $= c(c + 8) + 3(c + 8)$
*Factorise again
Consider $(c + 8) = \blacksquare$*
 $= \blacksquare(c + 8)$ $= (c + 8)(c + 3)$

b) Factorise $a^2 + 3a + 2a + 6$
 $= a(a + 3) + 2(a + 3)$
 $=$ $=$

c) Factorise $s^2 + 6s + 5s + 30$
 $=$
 $=$

d) Factorise $h^2 + 5h + 4h + 20$
 $=$
 $=$

e) Factorise $v^2 + 7v + 3v + 21$
 $=$
 $=$

f) Factorise $4n + n^2 + 16 + 4n$
 $=$
 $=$

g) Factorise $6t + t^2 - 42 - 7t$
 $=$
 $=$

h) Factorise $4b + 4 - b^2 - b$
 $=$
 $=$

i) Factorise $5p - 10 + p^2 - 2p$
 $=$
 $=$

j) Factorise $q^2 + 5q - 4q - 20$
 $=$
 $=$

Skill 19.8 Factorising using the difference of perfect squares.

MM9 11 22 33 44
MM10 11 22 33 44

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$$\begin{aligned}
 (a + b)(a - b) &= a \times a + a \times -b + b \times a + b \times -b \\
 &= a^2 - ab + ba - b^2 \\
 &= a^2 - b^2
 \end{aligned}$$

Diagram showing the expansion of $(a + b)(a - b)$ with arrows indicating the multiplication of terms: 1 (a × a), 2 (a × -b), 3 (b × a), 4 (b × -b). The result is $a^2 - ab + ba - b^2$, where $-ab$ and ba are grouped as like terms to simplify to $a^2 - b^2$.

Q. Factorise $5w^2 - 20$

A. $5w^2 - 20$

$= 5(w^2 - 4)$

$= 5(w^2 - 2^2)$

$= 5(w + 2)(w - 2)$

Take out the CF of 5

$4 = 2^2$

Use $a^2 - b^2 = (a + b)(a - b)$
where $a = w$ and $b = 2$

a) Factorise $c^2 - 81$

$b^2 = 81$
 $b = 9$

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2$

$= (c + 9)(c - 9)$

b) Factorise $y^2 - 4$

$=$

c) Factorise $d^2 - e^2$

$=$

d) Factorise $36 - h^2$

$=$

e) Factorise $4j^2 - 9$

$=$

f) Factorise $2c^2 - 50$

$=$

g) Factorise $p^2 - 81q^2$

$=$

h) Factorise $80 - 5y^2$

$=$

i) Factorise $9a^2 - 36b^2$

$=$

j) Factorise $75 - 3z^2$

$=$

k) Factorise $3d^2 - 27$

$=$

l) Factorise $100 - 4k^2$

$=$

- Write two sets of brackets. Because x^2 can only be produced from $x \times x$, write the factors of the squared pronumeral in the brackets $(x \quad)(x \quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factorise $x^2 - 9x + 8$

A. $x^2 - 9x + 8$

Write x in the brackets
 $= (x \quad)(x \quad)$
List the factors of +8
 $8 = 1 \times 8 = -1 \times -8 = 2 \times 4 = -2 \times -4$
Only -1x and -8x can make -9x
 $= x^2 - 1x - 8x + 8$
Determine the x terms
 $= (x - 1)(x - 8)$

AND/OR consider
 $= (x^2 - 1x) - (8x + 8)$ *Group 2 and 2*
 $= x(x - 1) - 8(x - 1)$
Factorise each group
 $= x(\blacksquare) - 8(\blacksquare)$
Factorise again Consider (x - 1) = \blacksquare
 $= (\blacksquare)(x - 8)$
 $= (x - 1)(x - 8)$

CHECK
Expand the brackets
 $= (x - 1)(x - 8)$
 $= x^2 - 1x - 8x + 8$
 $= x^2 - 9x + 8 = (x - 1)(x - 8) \checkmark$

a) Find the missing factor
 $x^2 + 7x + 10$

$10 = 2 \times 5 = -2 \times -5$

$5x + 2x = 7x = (x + 5)(x + 2)$

b) Find the missing factor
 $d^2 - 4d + 4$

$4 = 2 \times 2 = -2 \times -2$

$4d - 2d = 2d = (d - 2)(\quad)$

c) Find the missing factor
 $s^2 + 4s + 3$

$3s + s = 4s = (s + 3)(\quad)$

d) Find the missing factor
 $g^2 + 8g + 15$

$5g + 3g = 8g = (g + 5)(\quad)$

e) Factorise
 $m^2 + 2m - 24$

$-24 = -4 \times 6 = 4 \times -6$

$6m - 4m = 2m = (\quad)$

Which pair can be added to get +2m?

f) Factorise
 $j^2 + 11j + 24$

$8j + 3j = 11j = (\quad)$

g) Factorise
 $y^2 + 5y + 4$

$4y + y = 5y = (\quad)$

h) Factorise
 $z^2 - 6z + 8$

$8z - 4z = 4z = (\quad)$

i) Factorise
 $c^2 - 6c + 5$

$5c - c = 4c = (\quad)$

j) Factorise
 $p^2 - 6p - 16$

$4p - 20 = -16 = (\quad)$