

20. [Algebra - Equations]

Skill 20.1 Solving one-step equations by using the inverse operations of + and -

MM9 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation	Inverse Operation	Operation	Inverse Operation
+	-	-	+
$x + 3 = 6$ $x + 3 - 3 = 6 - 3$ $x = 3$		$x - 3 = 6$ $x - 3 + 3 = 6 + 3$ $x = 9$	

Q. Solve for x : $x - 7 = 4$

A. $x - 7 = 4$ Operation: -7
 $x - 7 + 7 = 4 + 7$ Inverse of -7 is $+7$
 Simplify: $-7 + 7 = 0$ $x = 11$

Operation: $+2$

a) Solve for x : $x + 2 = 5$

$$x + 2 - 2 = 5 - 2$$

$x =$

b) Solve for x : $x + 4 = 9$

$$x + 4 - 4 = 9 - 4$$

$x =$

c) Solve for x : $x + 6 = 9$

$x =$

d) Solve for x : $x + 4 = 2$

$x =$

e) Solve for x : $x + 7 = -3$

$x =$

f) Solve for x : $5 + x = 2$

$x =$

g) Solve for x : $x - 3 = 5$

$$x - 3 + 3 = 5 + 3$$

$x =$

h) Solve for x : $x - 8 = 4$

$$x - 8 + 8 = 4 + 8$$

$x =$

i) Solve for x : $x - 7 = 9$

$x =$

j) Solve for x : $x - 2 = 14$

$x =$

k) Solve for x : $x - 7 = 7$

$x =$

l) Solve for x : $x - 9 = 12$

$x =$

m) Solve for x : $8 - x = 4$

$x =$

n) Solve for x : $x - 5 = -7$

$x =$

o) Solve for x : $6 - x = 9$

$x =$

Skill 20.2 Solving one-step equations by using the inverse operations of \times and \div

MM9 11 22 33 44
MM10 11 22 33 44

- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation	Inverse Operation	Operation	Inverse Operation
\times	\div	\div	\times
$3x = 6$		$\frac{x}{3} = 6$	
$\frac{3x}{3} = \frac{6}{3}$		$\frac{x}{3} \times 3 = 6 \times 3$	
$x = 2$		$x = 18$	

Q. Solve for x : $\frac{x}{3} = 5$

A. $\frac{x}{3} = 5$ Operation: $\div 3$
 $\frac{x}{\cancel{3}} \times \cancel{3} = 5 \times 3$ Inverse of $\div 3$ is $\times 3$
 $x = 15$

a) Solve for x : $\frac{x}{7} = 4$ Operation: $\div 7$

b) Solve for x : $\frac{x}{3} = 3$

c) Solve for x : $\frac{x}{2} = 3$

$\frac{x}{\cancel{7}} \times \cancel{7} = 4 \times 7$

$x = 28$

$x =$

$x =$

d) Solve for x : $4x = 16$ Operation: $\times 4$

e) Solve for x : $3x = 12$

f) Solve for x : $2x = 14$

$\frac{1}{\cancel{4}} \cancel{4}x = \frac{16}{\cancel{4}}$ Inverse of $\times 4$ is $\div 4$ Simplify: $\div 4$

$x = 4$

$x =$

g) Solve for x : $\frac{x}{2} = 6$

h) Solve for x : $\frac{x}{5} = 2$

i) Solve for x : $\frac{x}{8} = 6$

$x =$

$x =$

$x =$

j) Solve for x : $3x = 27$

k) Solve for x : $4x = 28$

l) Solve for x : $5x = 45$

$x =$

$x =$

$x =$

Skill 20.3 Solving two-step equations by using the inverse operations of +, -, × and ÷

- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $5 + \frac{2x}{3} = 1$

A. $5 + \frac{2x}{3} = 1$

$5 - 5 + \frac{2x}{3} = 1 - 5$ *Inverse of + 5 is - 5*

$\frac{2x}{3} \times 3 = -4 \times 3$ *Inverse of ÷ 3 is × 3*

$\frac{2x}{2} = \frac{-12}{2}$ *Inverse of × 2 is ÷ 2*

$x = -6$

a) Solve for x: $4x - 1 = 11$

Inverse of - 1 is + 1

$4x - 1 + 1 = 11 + 1$

$4x = 12$

Inverse of × 4 is ÷ 4

$\frac{4x}{4} = \frac{12}{4}$

$x = \boxed{3}$

b) Solve for x: $7 + 3x = 22$

$7 - 7 + 3x = 22 - 7$

$3x =$

$=$

$=$

$x = \boxed{}$

c) Solve for x: $2x + 7 = -3$

$=$

$=$

$=$

$x = \boxed{}$

d) Solve for x: $5x - 1 = 24$

$=$

$=$

$=$

$x = \boxed{}$

e) Solve for x: $15 + 10x = 45$

$=$

$=$

$=$

$x = \boxed{}$

f) Solve for x: $3x + 12 = 3$

$=$

$=$

$=$

$x = \boxed{}$

g) Solve for x: $\frac{x}{4} + 3 = 5$

$\frac{x}{4} + 3 - 3 = 5 - 3$

$=$

$=$

$x = \boxed{}$

h) Solve for x: $\frac{x}{5} + 3 = 1$

$=$

$=$

$=$

$x = \boxed{}$

i) Solve for x: $\frac{5x}{2} - 3 = -1$

$=$

$=$

$=$

$x = \boxed{}$

Skill 20.4 Solving equations by first expanding the brackets.

MM9 11 22 33 44
MM10 11 22 33 44

- Expand the brackets.
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x : $6(2 - x) = -18$

A. $6(2 - x) = -18$ Expand the ()
 $12 - 6x = -18$
 $12 - 12 - 6x = -18 - 12$ Inverse of $+12$ is -12
 $-6x = -30$
 $\frac{-6x}{-6} = \frac{-30}{-6}$ Inverse of $\times -6$ is $\div -6$
 $x = 5$

a) Solve for x : $3(x - 2) = 12$

b) Solve for x : $3(2 + x) = 21$

c) Solve for x : $2(x - 3) = 14$

Expand the ()

$$3x - 6 = 12$$

Inverse of -6 is $+6$

$$3x - 6 + 6 = 12 + 6$$

Inverse of $\times 3$ is $\div 3$

$$\frac{3x}{3} = \frac{12+6}{3}$$

$$x = \boxed{4}$$

=

=

=

$$x = \boxed{}$$

=

=

=

$$x = \boxed{}$$

d) Solve for x : $5(1 + x) = 20$

e) Solve for x : $7(2 + x) = 35$

f) Solve for x : $4(x - 3) = 4$

=

=

=

$$x = \boxed{}$$

=

=

=

$$x = \boxed{}$$

=

=

=

$$x = \boxed{}$$

g) Solve for x : $4(x - 5) = 8$

h) Solve for x : $2(9 - x) = 8$

i) Solve for x : $3(2x - 3) = 15$

=

=

=

$$x = \boxed{}$$

=

=

=

$$x = \boxed{}$$

=

=

=

$$x = \boxed{}$$

- If necessary, expand the brackets. (see skill 20.4, page 196)
- Combine all variables on one side of the equation by using inverse operations.
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

<p>Q. Solve for x: $x = 3x + 12$</p>	<p>A.</p> $x = 3x + 12$ <p style="text-align: right; margin-right: 20px;">Combine x's: $-3x$</p> $x - 3x = 3x - 3x + 12$ $-2x = 12$ $\frac{-2x}{-2} = \frac{12}{-2}$ <p style="text-align: right; margin-right: 20px;">Inverse of $\times -2$ is $\div -2$</p> $x = -6$
--	--

<p>a) Solve for x: $6 - 3x = 3x$</p> $6 - 6 - 3x = 3x - 6$ $-3x - 3x = 3x - 3x - 6$ <p style="margin-left: 20px;">Inverse of $\times -6$ is $\div -6$</p> $\frac{-6x}{-6} = \frac{-6}{-6}$ $x = 1$	<p>b) Solve for x: $6x + 4 = 8x$</p> $=$ $=$ $=$ $=$ $x =$	<p>c) Solve for x: $25 - 4x = x$</p> $=$ $=$ $=$ $=$ $x =$
--	--	--

<p>d) Solve for x: $6x - 4 = 5x$</p> $=$ $=$ $=$ $x =$	<p>e) Solve for x: $15 - 3x = 2x$</p> $=$ $=$ $=$ $x =$	<p>f) Solve for x: $7x - 24 = 4x$</p> $=$ $=$ $=$ $x =$
--	---	---

<p>g) Solve for x:</p> <p style="margin-left: 20px;">Expand the ()</p> $4x + 2(3x - 4) = 22$ <p style="margin-left: 20px;">Combine the variables</p> $4x + 6x - 8 = 22$ $10x - 8 + 8 = 22 + 8$ $\frac{10x}{10} = \frac{30}{10}$ $x = 3$	<p>h) Solve for x:</p> $2x + 3(4x - 3) = 19$ $=$ $=$ $=$ $x =$	<p>i) Solve for x:</p> $2(x - 3) - 3x = -12$ $=$ $=$ $=$ $x =$
---	---	---

Skill 20.6 Solving equations involving fractions.

- Use inverse operations rules to isolate any fractions.
- Rewrite all expressions as fractions if necessary.
- Cross multiply. (see skill 10.6, page 96)
- Combine all variables on one side of the equation by using inverse operations. (see skill 20.5, page 197)
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

<p>Q. Solve for x: $\frac{x}{3} = x + 4$</p>	<p>A.</p> $\frac{x}{3} = x + 4$ $\frac{x}{3} \times \frac{x+4}{1} \quad \text{Cross multiply}$ $x = 3(x + 4)$ $x = 3x + 12$ $x - 3x = 3x - 3x + 12 \quad \text{Combine } x\text{'s: } -3x$ $-2x = 12$ $\frac{-2x}{-2} = \frac{12}{-2} \quad \text{Inverse of } \times -2 \text{ is } \div -2$ $x = -6$
--	---

a) Solve for x: $\frac{x}{4} - 10 = -x$

Isolate the fraction

$$\frac{x}{4} - 10 + 10 = -x + 10$$

$$\frac{x}{4} \times \frac{-x+10}{1} \quad \text{Rewrite expression as fraction}$$

$$x = 4(-x + 10)$$

$$x + 4x = -4x + 4x + 40$$

$$5x = 40$$

$$x = \boxed{}$$

b) Solve for x: $\frac{18}{x} = 2$

=

=

=

$$x = \boxed{}$$

c) Solve for x: $\frac{20 - 2x}{3} = 2$

=

=

=

$$x = \boxed{}$$

d) Solve for x: $\frac{2x}{5} = x - 3$

=

=

=

=

$$x = \boxed{}$$

e) Solve for x: $\frac{x+4}{3} = \frac{10-x}{4}$

=

=

=

=

$$x = \boxed{}$$

f) Solve for x: $\frac{2x}{3} + 10 = 4x$

=

=

=

=

$$x = \boxed{}$$

- Manipulate the inequation in the same way as you would an equation.

Except:

- When both sides are multiplied or divided by a negative number, reverse the inequality signs.
< becomes > and ≤ becomes ≥.

<p>Q. Solve the inequation: $4x - 7 \leq 5$</p>	<p>A.</p> $4x - 7 \leq 5$ $4x \not\cancel{-7} \not\cancel{+7} \leq 5 + 7$ $\not\cancel{4}x \leq \not\cancel{12}^3$ $\not\cancel{4}^1 \leq \not\cancel{4}^1$ $x \leq 3$
---	---

<p>a) Solve the inequation: $20 \geq 5(7 - 2x) - 35$</p> <p>$20 \geq \cancel{35} - 10x - \cancel{35}$</p> <p>$\cancel{20} - \cancel{20} + 10x \geq -\cancel{10}x + \cancel{10}x - 20$</p> <p>$10x \geq -20$</p> <p>$\frac{10x}{10} \geq \frac{-20}{10}$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">$x \geq -2$</div>	<p>b) Solve the inequation: $3x - 8 < 7$</p> <p>$<$</p> <p>$<$</p> <p>$<$</p> <p>$<$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">$x <$</div>	<p>c) Solve the inequation: $2x + 6 \leq 10$</p> <p>\leq</p> <p>\leq</p> <p>\leq</p> <p>\leq</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>
---	--	--

<p>d) Solve the inequation: $12 - x > 2(x + 3)$</p> <p>$12 - x > 2x + 6$</p> <p>$\cancel{12} - \cancel{12} - x > 2x + 6 - 12$</p> <p>$-x - 2x > \cancel{2x} - \cancel{2x} - 6$</p> <p>$-3x > -6$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; display: inline-block; margin-left: 10px;">Both sides negative so reverse inequality sign</div> <p>$\frac{-3x}{\cancel{-3}} < \frac{-6}{\cancel{-3}}$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">$x < 2$</div>	<p>e) Solve the inequation: $-5(x + 7) \geq 10$</p> <p>\geq</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>	<p>f) Solve the inequation: $4 < 2(3 - 2x) - 10$</p> <p>$<$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>
---	--	--

<p>g) Solve the inequation: $\frac{3(x + 4)}{2} > 15$</p> <p>$>$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>	<p>h) Solve the inequation: $\frac{4(x + 1)}{4} \geq 10$</p> <p>$\geq$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>	<p>i) Solve the inequation: $\frac{4x}{3} - x > -1$</p> <p>$>$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"></div>
---	---	---

EITHER

- Find the value of one of the variables in relation to the other.
- Substitute this value of the variable into the other equation.
- Solve for one variable.
- Substitute the result into either equation to find the second variable.

OR

- Add or subtract the equations together to eliminate one of the variables.

Q. Solve the simultaneous equations:

$$2x + 3y = 3$$

$$x + 3y = 6$$

A. $2x + 3y = 3$ (1)
 $x + 3y = 6$ (2)
 $x = -3$
 $-3 + 3y = 6$
 $-3 + 3 + 3y = 6 + 3$
 $3y = 9$
 $\frac{3y}{3} = \frac{9}{3}$
 $y = 3$
 $(-3, 3)$

Eliminate 'y' by subtracting (1) - (2)

Substitute $x = -3$ into (2)

a) Solve the simultaneous equations:

$$y = 3x - 9$$

$$x = 4$$

Substitute $x = 4$ into (1)

$y = 3 \times 4 - 9$

 $y = 12 - 9$

 $y = 3$ **(4,3)**

b) Solve the simultaneous equations:

$$x + y = 5$$

$$y = x + 1$$

Substitute $y = x + 1$ into (1)

.....

c) Solve the simultaneous equations:

$$y = 2x + 1$$

$$y = 3x - 2$$

.....

d) Solve the simultaneous equations:

$$4 = 2x + y$$

$$x - 5 = y$$

.....

e) Solve the simultaneous equations:

$$x + y = 1$$

$$x - y = 3$$

.....

f) Solve the simultaneous equations:

$$x - y = 2$$

$$3x + y = 14$$

.....

- Factorise the expression. (see skills 19.4, page 187 and skill 19.7, page 190)
- Make either factor equal zero. Use the zero multiplication property.
 $a \times 0 = 0$ and $0 \times a = 0$

Q. Solve for x :
 $x^2 + 7x = 0$

A. $x^2 + 7x = 0$

Factorise

$x(x + 7) = 0$

$x = 0$

OR

$x + 7 - 7 = 0 - 7$

$x = -7$

0, -7

If either
 $x = 0$
or $(x + 7) = 0$
then $x^2 + 7x = 0$

a) Solve for x :
 $x^2 - 16 = 0$

Factorise

If either
 $(x + 4) = 0$
or $(x - 4) = 0$
then $x^2 - 16 = 0$

$(x + 4)(x - 4) = 0$

so $x = 4$ or $x = -4$

4, -4

b) Solve for x :
 $x^2 - 4 = 0$

c) Solve for x :
 $x^2 - 2x = 0$

d) Solve for x :
 $x^2 - 3x = 0$

e) Solve for x :
 $x^2 + 4x = 0$

f) Solve for x :
 $x^2 + 5x = 0$

g) Solve for x :
 $x^2 - 64 = 0$

h) Solve for x :
 $x^2 - 144 = 0$