

23. [Angles]

Skill 23.1 Choosing the correct terms related to angles.

MM9 1 22 33 44
MM10 11 22 33 44

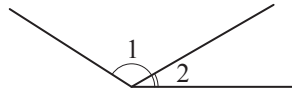
- Consider the definitions and properties of a variety of angles.
(see Glossary or Maths Facts, page 431)

Hints: An angle can be classified according to its size (acute, right, obtuse, straight and reflex).

Two angles can be classified according to their position in relation to one another (adjacent, supplementary, complementary or vertically opposite).

Q. Which would best describe the angles marked in this diagram?

- A) vertically opposite
- B) supplementary
- C) adjacent

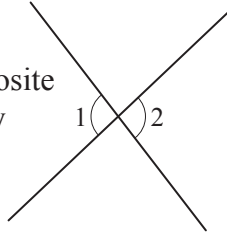


- A.** A) vertically opposite \Rightarrow equal angles
(1 and 2 are not equal) *false*
B) supplementary \Rightarrow angles add to 180°
(1 and 2 add to less than 180°) *false*
C) adjacent \Rightarrow angles share the vertex
and an arm *true*

The answer is **C**

a) Which would best describe the angles marked in this diagram?

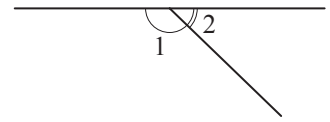
- A) right
- B) vertically opposite
- C) supplementary



B

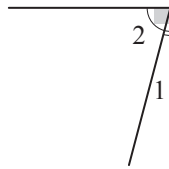
b) Which would best describe the angles marked in this diagram?

- A) straight
- B) supplementary
- C) acute



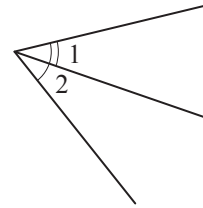
c) Which would best describe the angles marked in this diagram?

- A) reflex
- B) right
- C) complementary



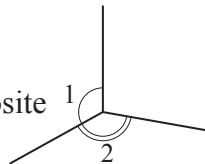
d) Which would best describe the angles marked in this diagram?

- A) acute
- B) obtuse
- C) complementary



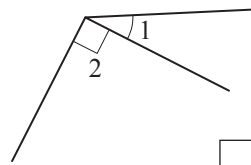
e) Which would best describe the angles marked in this diagram?

- A) supplementary
- B) obtuse
- C) vertically opposite



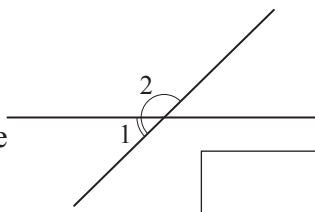
f) Which would best describe the angles marked in this diagram?

- A) complementary
- B) right
- C) adjacent



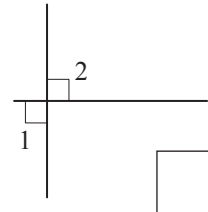
g) Which would best describe the angles marked in this diagram?

- A) supplementary
- B) acute
- C) vertically opposite



h) Which would best describe the angles marked in this diagram?

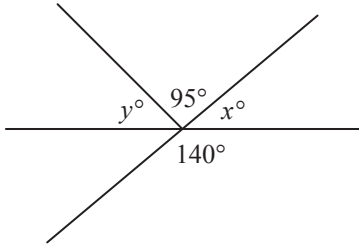
- A) straight
- B) complementary
- C) right



Skill 23.2 Finding the complement and the supplement of a given angle.

- Use the properties:
 - the sum of two complementary angles is 90° .
 - the sum of two supplementary angles is 180° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the values of x° and y° .



A. x° and 140° are supplementary:

$$x^\circ + 140^\circ = 180^\circ$$

$$x^\circ + 140^\circ - 140^\circ = 180^\circ - 140^\circ$$

$$x^\circ = 40^\circ$$

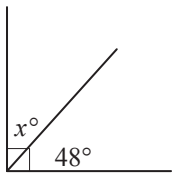
y° , 95° and x° are supplementary:

$$y^\circ + 95^\circ + 40^\circ = 180^\circ$$

$$y^\circ + 135^\circ - 135^\circ = 180^\circ - 135^\circ$$

$$y^\circ = 45^\circ$$

a) Find the value of x° .

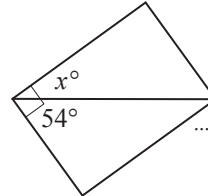


$$x^\circ + 48^\circ = 90^\circ$$

$$x^\circ + 48^\circ - 48^\circ = 90^\circ - 48^\circ$$

$$x^\circ = \boxed{42^\circ}$$

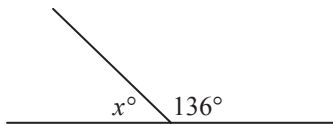
b) Find the value of x° .



$$x^\circ + 54^\circ = 90^\circ$$

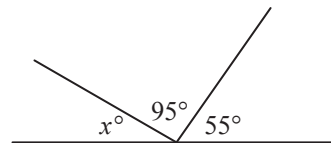
$$x^\circ = \boxed{}$$

c) Find the value of x° .



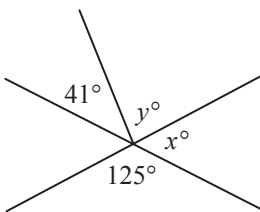
$$x^\circ = \boxed{}$$

d) Find the value of x° .



$$x^\circ = \boxed{}$$

e) Find the values of x° and y° .

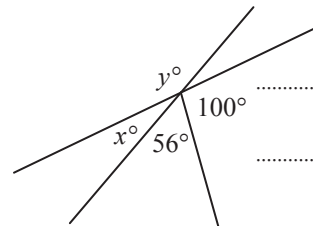


$$x^\circ + 125^\circ = 180^\circ$$

$$x^\circ = \boxed{}$$

$$y^\circ = \boxed{}$$

f) Find the values of x° and y° .



$$x^\circ + 156^\circ = 180^\circ$$

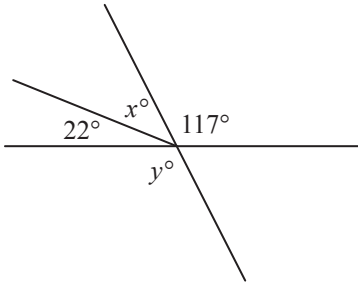
$$x^\circ = \boxed{}$$

$$y^\circ = \boxed{}$$

Skill 23.3 Working with vertically opposite angles.

- Use the definition of vertically opposite angles. (see Glossary, page 421)
- Consider complementary and supplementary angles. (see skill 23.2, page 237)

Q. Find the values of x° and y° .



A. y° and 117° are vertically opposite:

$$y^\circ = 117^\circ$$

x° , 22° and 117° are supplementary:

$$x^\circ + 22^\circ + 117^\circ = 180^\circ$$

$$x^\circ + 139^\circ - 139^\circ = 180^\circ - 139^\circ$$

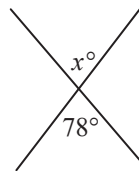
$$x^\circ = 41^\circ$$

a) Find the value of x° .



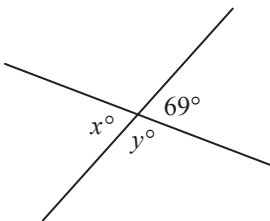
$x^\circ =$

b) Find the value of x° .



$x^\circ =$

c) Find the values of x° and y° .



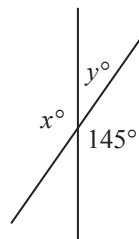
$x^\circ =$

$$y^\circ + 69^\circ = 180^\circ$$

$$y^\circ + 69^\circ - 69^\circ = 180^\circ - 69^\circ$$

$y^\circ =$

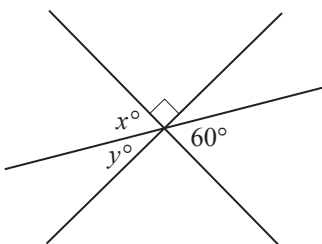
d) Find the values of x° and y° .



$x^\circ =$

$y^\circ =$

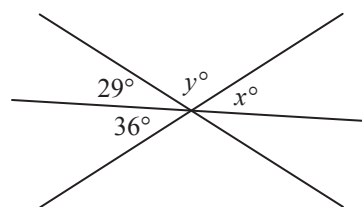
e) Find the values of x° and y° .



$x^\circ =$

$y^\circ =$

f) Find the values of x° and y° .

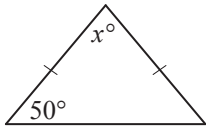


$x^\circ =$

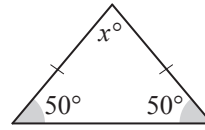
$y^\circ =$

- Use the property:
- the sum of the interior angles of any triangle is 180° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.



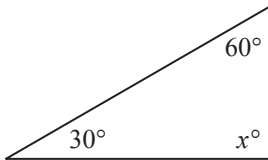
Isosceles triangle \Rightarrow base angles are equal

$$x^\circ + 50^\circ + 50^\circ = 180^\circ$$

$$x^\circ + 100^\circ - 100^\circ = 180^\circ - 100^\circ$$

$$x^\circ = 80^\circ$$

a) Find the value of x° .

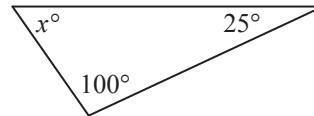


$$x^\circ + 90^\circ = 180^\circ$$

$$x^\circ + 90^\circ - 90^\circ = 180^\circ - 90^\circ$$

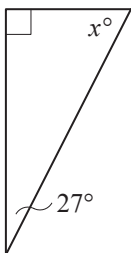
$$x^\circ = \boxed{}$$

b) Find the value of x° .



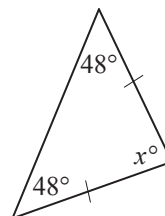
$$x^\circ = \boxed{}$$

c) Find the value of x° .



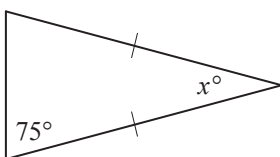
$$x^\circ = \boxed{}$$

d) Find the value of x° .



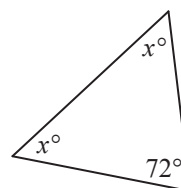
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

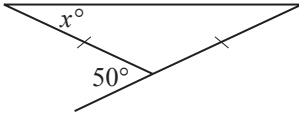
f) Find the value of x° .



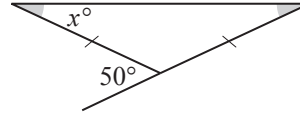
$$x^\circ = \boxed{}$$

- Use the property:
 - an exterior angle to a triangle is equal to the sum of the two opposite interior angles of the triangle.
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.



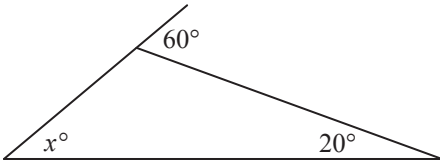
Isosceles triangle \Rightarrow base angles are equal

$$x^\circ + x^\circ = 50^\circ$$

$$2x^\circ \div 2 = 50^\circ \div 2$$

$$x^\circ = 25^\circ$$

a) Find the value of x° .

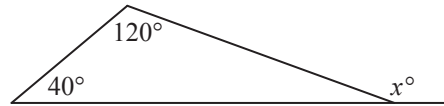


$$x^\circ + 20^\circ = 60^\circ$$

$$x^\circ + 20^\circ - 20^\circ = 60^\circ - 20^\circ$$

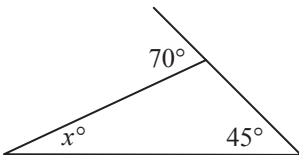
$$x^\circ = \boxed{}$$

b) Find the value of x° .



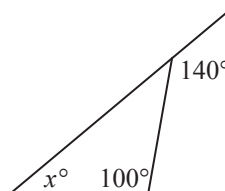
$$x^\circ = \boxed{}$$

c) Find the value of x° .



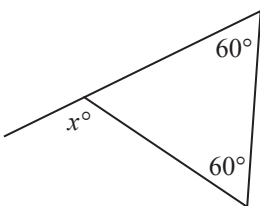
$$x^\circ = \boxed{}$$

d) Find the value of x° .



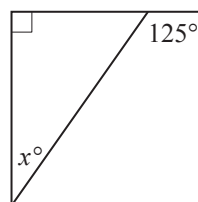
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

f) Find the value of x° .



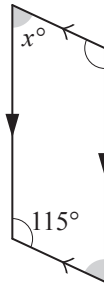
$$x^\circ = \boxed{}$$

- Use the property:
- the sum of the interior angles of any quadrilateral is 360° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.



Parallelogram \Rightarrow opposite angles are equal

$$2x^\circ + 2 \times 115^\circ = 360^\circ$$

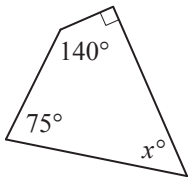
$$2x^\circ + 230^\circ - 230^\circ = 360^\circ - 230^\circ$$

$$2x^\circ = 130^\circ$$

$$2x^\circ \div 2 = 130^\circ \div 2$$

$$x^\circ = 65^\circ$$

a) Find the value of x° .

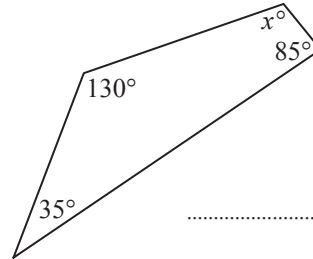


$$x^\circ + 90^\circ + 140^\circ + 75^\circ = 360^\circ$$

$$x^\circ + 305^\circ - 305^\circ = 360^\circ - 305^\circ$$

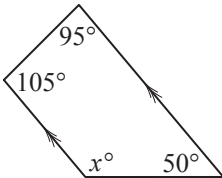
$$x^\circ =$$

b) Find the value of x° .



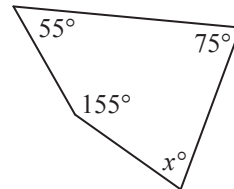
$$x^\circ =$$

c) Find the value of x° .



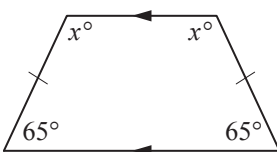
$$x^\circ =$$

d) Find the value of x° .



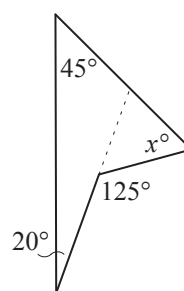
$$x^\circ =$$

e) Find the value of x° .



$$x^\circ =$$

f) Find the value of x° .



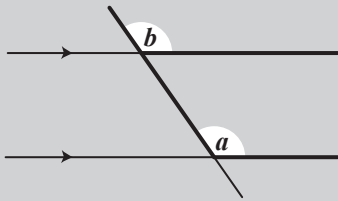
$$x^\circ =$$

Skill 23.7 Working with pairs of alternate, co-interior and corresponding angles.

MM9 11 22 33 44
MM10 11 22 33 44

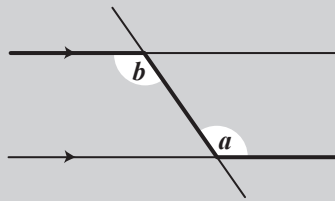
- Consider the classification and properties of the angles formed by intersecting a pair of parallel lines by a transversal. (see Glossary, pages 360, 365 and 368)

CORRESPONDING ANGLES



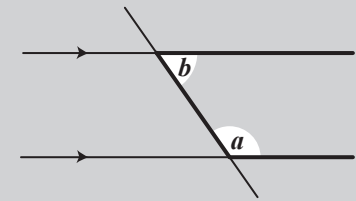
$\angle a = \angle b$

ALTERNATE ANGLES



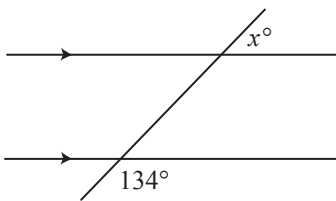
$\angle a = \angle b$

CO-INTERIOR ANGLES

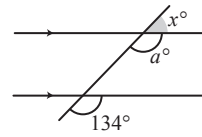


$\angle a + \angle b = 180^\circ$

Q. Find the value of x° .



A.



x° and a° are not in any category above

a° and 134° are corresponding angles

$\Rightarrow a^\circ = 134^\circ$

x° and a° are supplementary angles

$\Rightarrow x^\circ + a^\circ = 180^\circ$

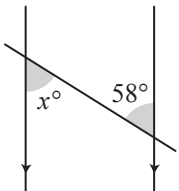
Substitute $a^\circ = 134^\circ \Rightarrow$ the equation becomes:

$x^\circ + 134^\circ = 180^\circ$

$x^\circ + 134^\circ - 134^\circ = 180^\circ - 134^\circ$

$x^\circ = 46^\circ$

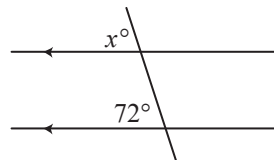
a) Find the value of x° .



alternate angles

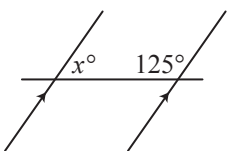
$\Rightarrow x^\circ =$

b) Find the value of x° .



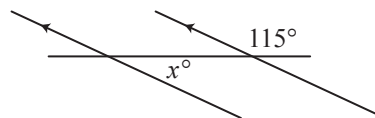
$\Rightarrow x^\circ =$

c) Find the value of x° .



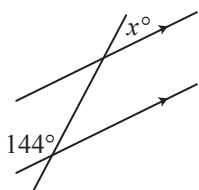
$\Rightarrow x^\circ =$

d) Find the value of x° .



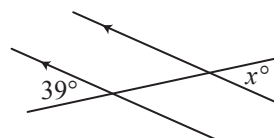
$\Rightarrow x^\circ =$

e) Find the value of x° .



$\Rightarrow x^\circ =$

f) Find the value of x° .

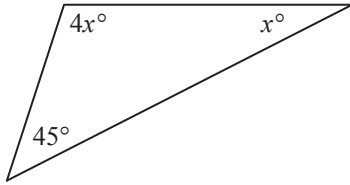


$\Rightarrow x^\circ =$

Skill 23.8 Finding the value of an angle in a variety of diagrams.

- Use the properties of angles. (see skills 23.1 to 23.7, pages 236 to 242 and Maths Facts, page 431)
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.

$$4x^\circ + x^\circ + 45^\circ = 180^\circ$$

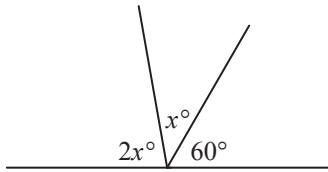
$$5x^\circ + 45^\circ - 45^\circ = 180^\circ - 45^\circ$$

$$5x^\circ = 135^\circ$$

$$5x^\circ \div 5^\circ = 135^\circ \div 5^\circ$$

$$x^\circ = 27^\circ$$

a) Find the value of x° .

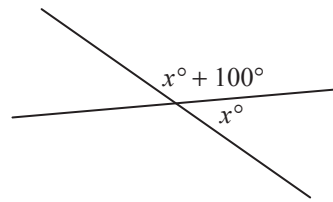


$$3x^\circ + 60^\circ - 60^\circ = 180^\circ - 60^\circ$$

$$3x^\circ \div 3 = 120^\circ \div 3$$

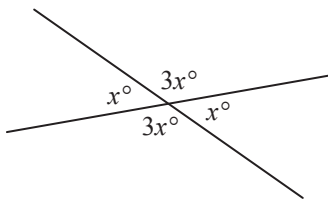
$$x^\circ = \boxed{}$$

b) Find the value of x° .



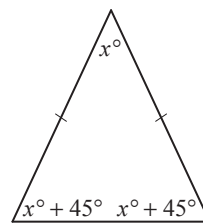
$$x^\circ = \boxed{}$$

c) Find the value of x° .



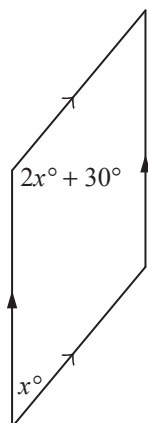
$$x^\circ = \boxed{}$$

d) Find the value of x° .



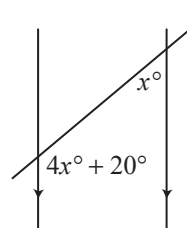
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

f) Find the value of x° .



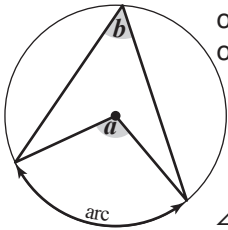
$$x^\circ = \boxed{}$$

Skill 23.9 Finding the value of an angle in a circle (1).

- Use the following properties of angles in circles:

Property 1

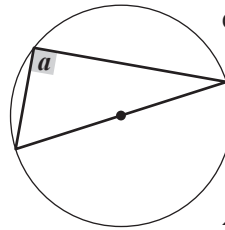
The angle that an arc forms at the centre of a circle is twice the size of the angle formed by the same arc on the circumference.



$$\angle a = 2 \times \angle b$$

Property 2

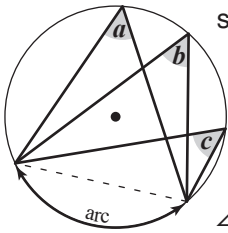
The angle formed on the circumference from a diameter of a circle is always a right angle.



$$\angle a = 90^\circ$$

Property 3

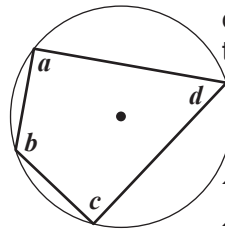
All angles at the circumference standing on the same arc, in the same segment, are equal.



$$\angle a = \angle b = \angle c$$

Property 4

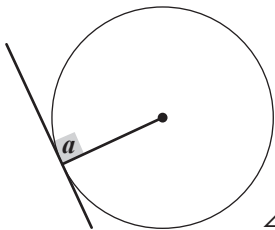
The opposite angles in a cyclic quadrilateral (all 4 vertices are on the circumference) add up to 180° (are supplementary).



$$\begin{aligned} \angle a + \angle c &= 180^\circ \\ \angle b + \angle d &= 180^\circ \end{aligned}$$

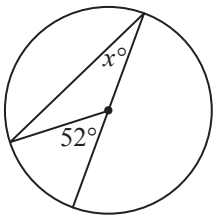
Property 5

Any tangent drawn on a circle meets the radius of the circle at right angles.



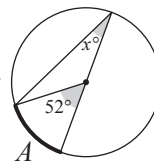
$$\angle a = 90^\circ$$

Q. Find the value of x° .



A.

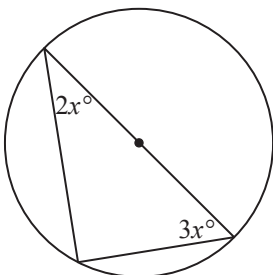
use property 1



52° formed at the centre by arc A
 x° formed on the circumference by arc A $\Rightarrow 2x^\circ = 52^\circ$
Solve the equation:

$$\begin{aligned} 2x^\circ &= 52^\circ \\ 2x^\circ \div 2 &= 52^\circ \div 2 \\ x^\circ &= \mathbf{26^\circ} \end{aligned}$$

a) Find the value of x° .



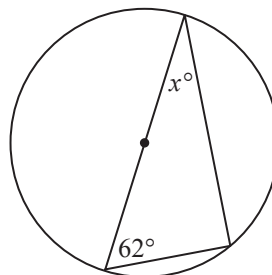
use property 2

$$2x^\circ + 3x^\circ = 90^\circ$$

$$5x^\circ \div 5 = 90^\circ \div 5$$

$$x^\circ = \boxed{}$$

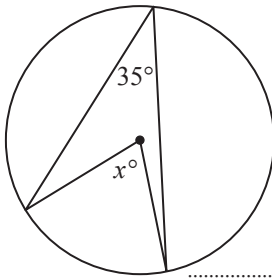
b) Find the value of x° .



$$x^\circ = \boxed{}$$

Skill 23.9 Finding the value of an angle in a circle (2).

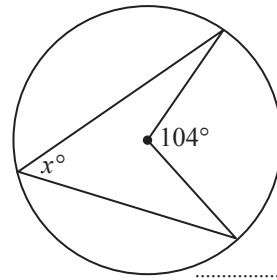
c) Find the value of x° .



$$x^\circ = 2 \times 35^\circ$$

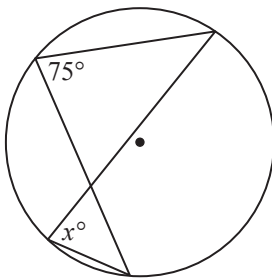
$$x^\circ =$$

d) Find the value of x° .



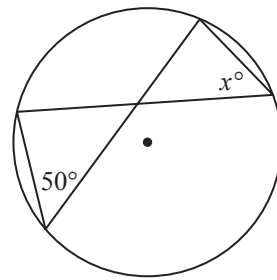
$$x^\circ =$$

e) Find the value of x° .



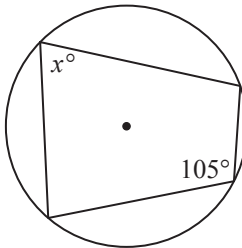
$$x^\circ =$$

f) Find the value of x° .



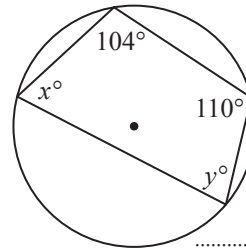
$$x^\circ =$$

g) Find the value of x° .



$$x^\circ =$$

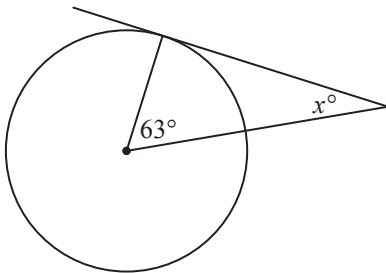
h) Find the values of x° and y° .



$$x^\circ =$$

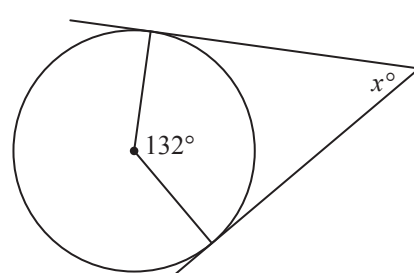
$$y^\circ =$$

i) Find the value of x° .



$$x^\circ =$$

j) Find the value of x° .



$$x^\circ =$$