27. [Angles]

Skill 27.1 Choosing the correct terms related to angles.

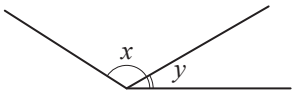
MM9 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Consider the definitions and properties of a variety of angles.
  (see Glossary or Maths Facts, page 455)

  Hints: An angle can be classified according to its size (acute, right, obtuse, straight and reflex). Two angles can be classified according to their position in relation to one another (adjacent, supplementary, complementary or vertically opposite).

Q. Which would describe the pair of angles marked x and y in this diagram?

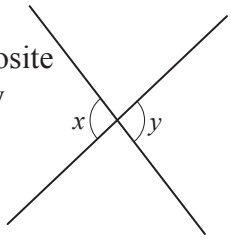
A) vertically opposite
B) supplementary
C) adjacent



A. *A) vertically opposite ⇒ equal angles (x and y are not equal) false*
B) supplementary ⇒ angles add to 180° (x and y add to less than 180°) false
C) adjacent ⇒ angles share the vertex and an arm true
*The answer is **C**.*

a) Which would describe the pair of angles marked x and y in this diagram?

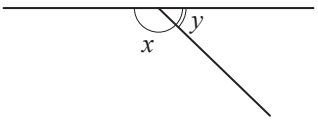
A) right
B) vertically opposite
C) supplementary



B

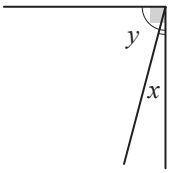
b) Which would describe the pair of angles marked x and y in this diagram?

A) straight
B) supplementary
C) acute



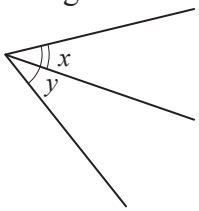
c) Which would describe the pair of angles marked x and y in this diagram?

A) reflex
B) right
C) complentary



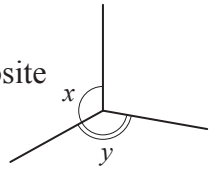
d) Which would describe the pair of angles marked x and y in this diagram?

A) acute
B) obtuse
C) complementary



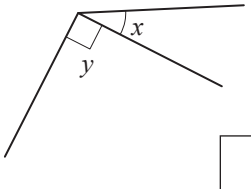
e) Which would describe the pair of angles marked x and y in this diagram?

A) supplementary
B) obtuse
C) vertically opposite



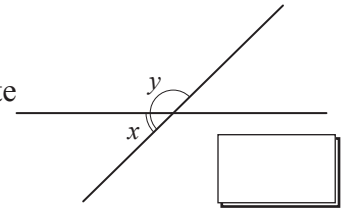
f) Which would describe the pair of angles marked x and y in this diagram?

A) complementary
B) right
C) adjacent



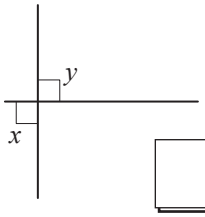
g) Which would describe the pair of angles marked x and y in this diagram?

A) supplementary
B) acute
C) vertically opposite



h) Which would describe the pair of angles marked x and y in this diagram?

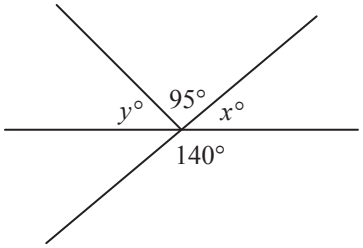
A) straight
B) complementary
C) right



Skill 27.2 Finding the complement and the supplement of a given angle.

- Use the properties:
  - the sum of two complementary angles is 90°.
  - the sum of two supplementary angles is 180°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the values of $x°$ and $y°$.



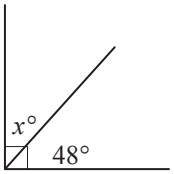
A. *$x°$ and 140° are supplementary:*

$$x° + 140° = 180°$$
$$x° + 140° - 140° = 180° - 140°$$
$$x° = \mathbf{40°}$$

$y°$, 95° and $x°$ are supplementary:

$$y° + 95° + 40° = 180°$$
$$y° + 135° - 135° = 180° - 135°$$
$$y° = \mathbf{45°}$$

a) Find the value of $x°$.

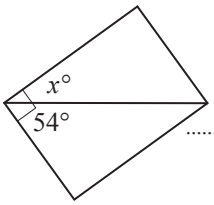


$x° + 48° = 90°$

$x° + 48° - 48° = 90° - 48°$

$x° =$ ☐

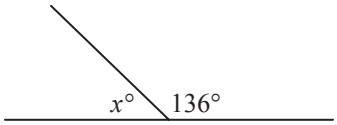
b) Find the value of $x°$.



$x° + 54° = 90°$

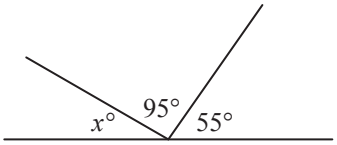
$x° =$ ☐

c) Find the value of $x°$.



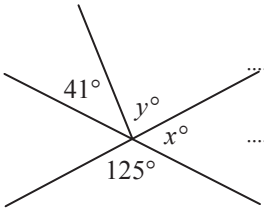
$x° =$ ☐

d) Find the value of $x°$.



$x° =$ ☐

e) Find the values of $x°$ and $y°$.

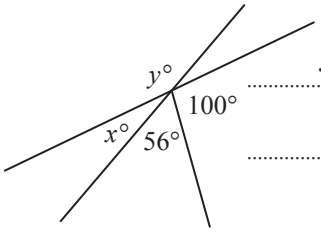


$x° + 125° = 180°$

$x° =$ ☐

$y° =$ ☐

f) Find the values of $x°$ and $y°$.



$x° + 156° = 180°$

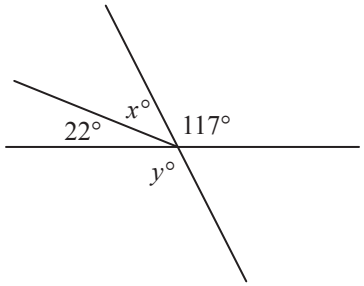
$x° =$ ☐

$y° =$ ☐

Skill 27.3 Working with vertically opposite angles.

- Use the definition of vertically opposite angles. (see Glossary, page 443 and Maths facts page 455)
- Consider complementary and supplementary angles. (see skill 27.2, page 326)

Q. Find the values of $x°$ and $y°$.



A. *$y°$ and $117°$ are vertically opposite:*

$y° = \mathbf{117°}$

$x°$, $22°$ and $117°$ are supplementary:

$x° + 22° + 117° = 180°$

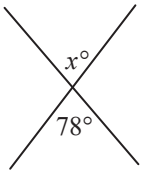
$x° + 139° - 139° = 180° - 139°$

$x° = \mathbf{41°}$

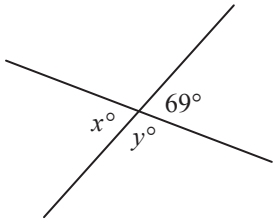
a) Find the value of $x°$.

26°

b) Find the value of $x°$.



c) Find the values of $x°$ and $y°$.



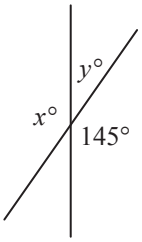
$x° =$ 69°

$y° + 69° = 180°$

$y° + 69° - 69° = 180° - 69°$

$y° =$

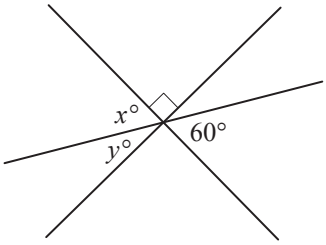
d) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

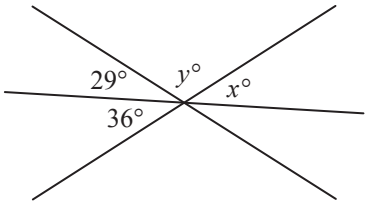
e) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

f) Find the values of $x°$ and $y°$.



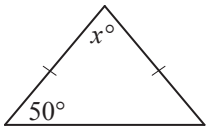
$x° =$

$y° =$

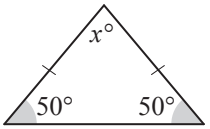
Skill 27.4 Working with angles in a triangle.

- Use the property:
  - the sum of the interior angles of any triangle is 180°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



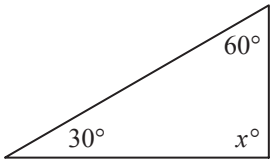
Isosceles triangle ⇒ base angles are equal

$x° + 50° + 50° = 180°$

$x° + 100° - 100° = 180° - 100°$

$x° = \mathbf{80°}$

a) Find the value of $x°$.

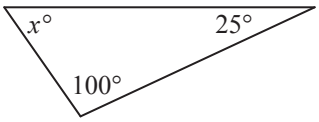


$x° + 90° = 180°$

$x° + 90° - 90° = 180° - 90°$

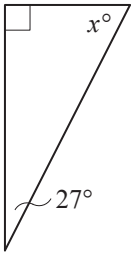
$x° =$ ☐

b) Find the value of $x°$.



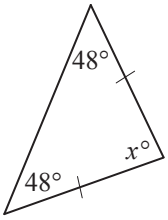
$x° =$ ☐

c) Find the value of $x°$.



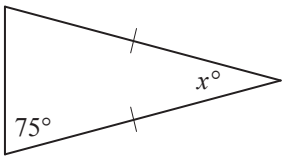
$x° =$ ☐

d) Find the value of $x°$.



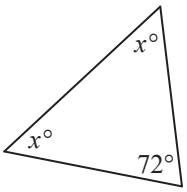
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

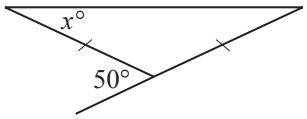


$x° =$ ☐

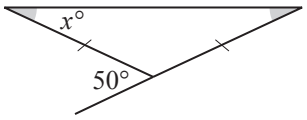
Skill 27.5 Finding the exterior angle of a triangle.

- Use the property:
  - an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



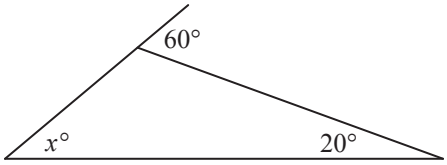
Isosceles triangle ⇒ base angles are equal

$x° + x° = 50°$

$2x° \div 2 = 50° \div 2$

$x° = \mathbf{25°}$

a) Find the value of $x°$.

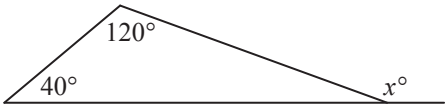


$x° + 20° = 60°$

$x° + 20° - 20° = 60° - 20°$

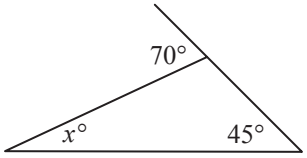
$x° =$ ☐

b) Find the value of $x°$.



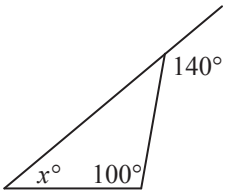
$x° =$ ☐

c) Find the value of $x°$.



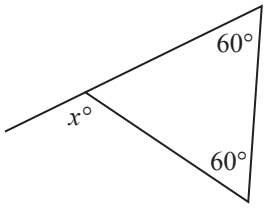
$x° =$ ☐

d) Find the value of $x°$.



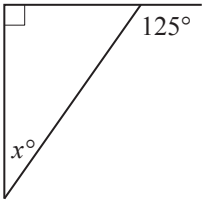
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

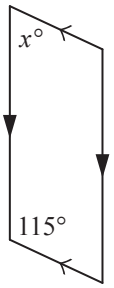


$x° =$ ☐

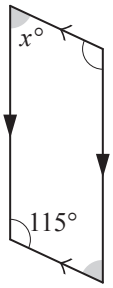
Skill 27.6 Working with angles in a quadrilateral.

- Use the property:
  - the sum of the interior angles of any quadrilateral is 360°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



Parallelogram ⇒ opposite angles are equal

$2x° + 2 \times 115° = 360°$

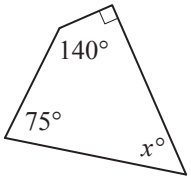
$2x° + 230° - 230° = 360° - 230°$

$2x° = 130°$

$2x° \div 2 = 130° \div 2$

$x° = \mathbf{65°}$

a) Find the value of $x°$.

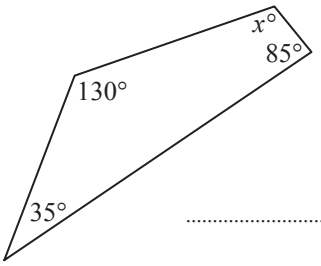


$x° + 90° + 140° + 75° = 360°$

$x° + 305° - 305° = 360° - 305°$

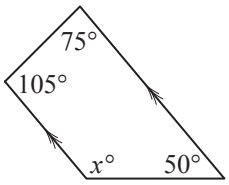
$x° =$ ☐

b) Find the value of $x°$.



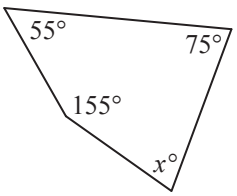
$x° =$ ☐

c) Find the value of $x°$.



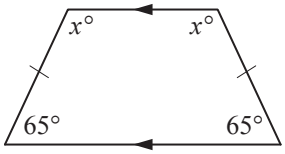
$x° =$ ☐

d) Find the value of $x°$.



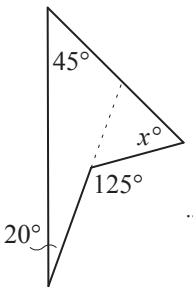
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

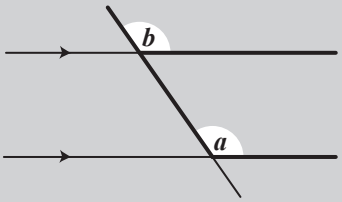


$x° =$ ☐

Skill 27.7 Working with pairs of alternate, co-interior and corresponding angles.

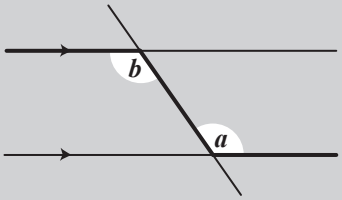
- Consider the classification and properties of the angles formed by intersecting a pair of parallel lines by a transversal. (see Glossary, pages 382, 387 and 390)

CORRESPONDING ANGLES



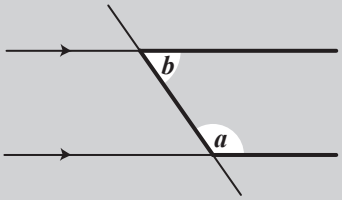
$\angle a = \angle b$

ALTERNATE ANGLES



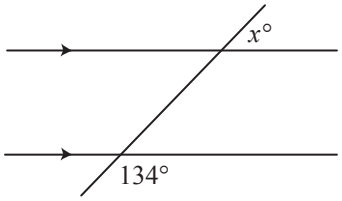
$\angle a = \angle b$

CO-INTERIOR ANGLES

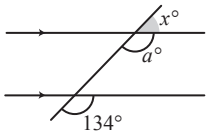


$\angle a + \angle b = 180°$

Q. Find the value of $x°$.



A.



x° and 134° are not in any category above

a° and 134° are corresponding angles

$\Rightarrow a° = 134°$

x° and a° are supplementary angles

$\Rightarrow x° + a° = 180°$

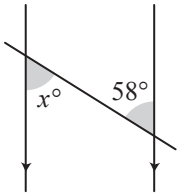
Substitute $a° = 134° \Rightarrow$ *the equation becomes:*

$x° + 134° = 180°$

$x° + 134° - 134° = 180° - 134°$

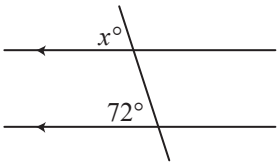
$x° = \mathbf{46°}$

a) Find the value of $x°$.



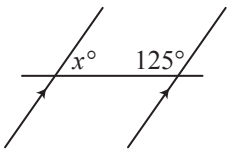
alternate angles $\Rightarrow x° =$ ☐

b) Find the value of $x°$.



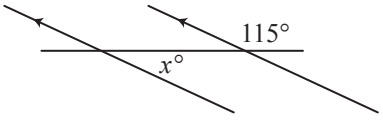
$\Rightarrow x° =$ ☐

c) Find the value of $x°$.



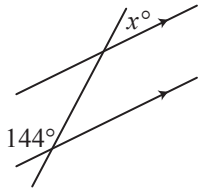
$\Rightarrow x° =$ ☐

d) Find the value of $x°$.



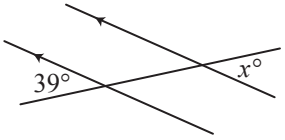
$\Rightarrow x° =$ ☐

e) Find the value of $x°$.



$\Rightarrow x° =$ ☐

f) Find the value of $x°$.

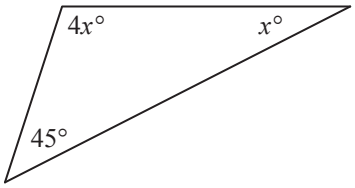


$\Rightarrow x° =$ ☐

Skill 27.8 Finding the value of an angle in a variety of diagrams.

- Use the properties of angles. (see skills 27.1 to 27.7, pages 325 to 331 and Maths Facts, page 455)
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

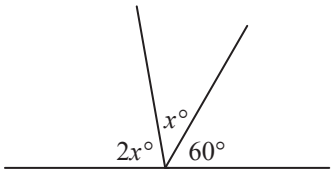
Q. Find the value of $x°$.



A.

$$4x° + x° + 45° = 180°$$
$$5x° + 45° - 45° = 180° - 45°$$
$$5x° = 135°$$
$$5x° \div 5° = 135° \div 5°$$
$$x° = \mathbf{27°}$$

a) Find the value of $x°$.

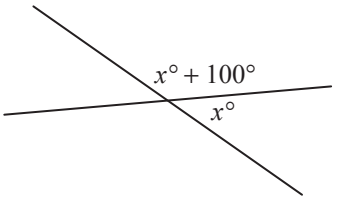


$3x° + 60° - 60° = 180° - 60°$

$3x° \div 3 = 120° \div 3$

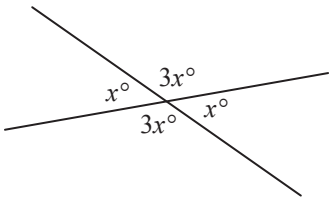
$x° =$ ☐

b) Find the value of $x°$.



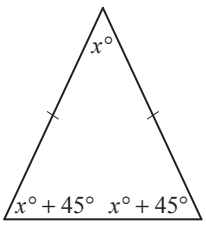
$x° =$ ☐

c) Find the value of $x°$.



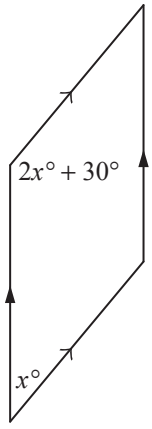
$x° =$ ☐

d) Find the value of $x°$.



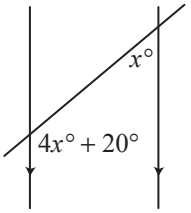
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.



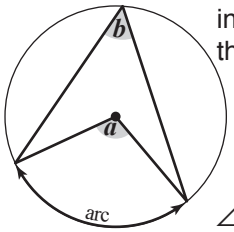
$x° =$ ☐

continues on page 334

Skill 27.9 Finding the value of an angle in a circle (1).

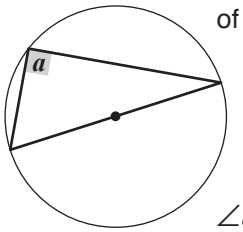
- Use the following properties of angles in circles:

Property 1 The angle at the centre of a circle is twice the size of the inscribed angle which intersects the same arc of the circle.



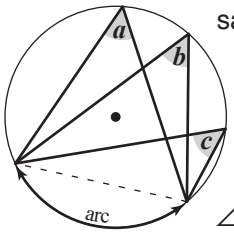
$\angle a = 2 \times \angle b$

Property 2 The angle formed on the circumference from a diameter of a circle is always a right angle.



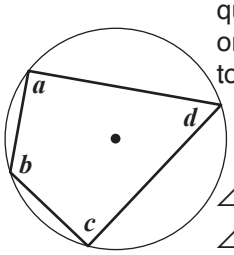
$\angle a = 90°$

Property 3 All angles at the circumference standing on the same arc, in the same segment, are equal.



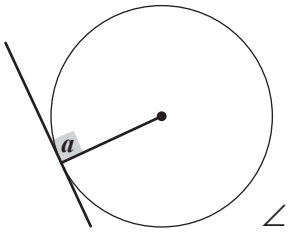
$\angle a = \angle b = \angle c$

Property 4 The opposite angles in a cyclic quadrilateral (all 4 vertices are on the circumference) add up to 180° (are supplementary).



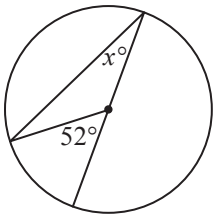
$\angle a + \angle c = 180°$
$\angle b + \angle d = 180°$

Property 5 Any tangent drawn on a circle meets the radius of the circle at right angles.

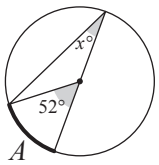


$\angle a = 90°$

Q. Find the value of $x°$.



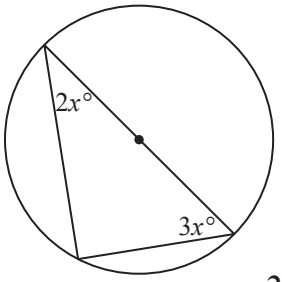
A. use property 1



52° is an angle at the centre intercepting arc A
x° is an inscribed angle intercepting arc A
$A \Rightarrow 2x° = 52°$
Solve the equation:

$2x° = 52°$
$2x° \div 2 = 52° \div 2$
$x° = \mathbf{26°}$

a) Find the value of $x°$.



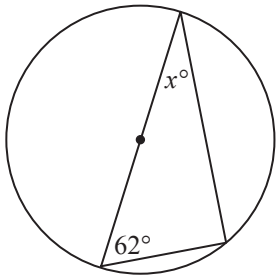
use property 2

$2x° + 3x° = 90°$

$5x° \div 5 = 90° \div 5$

$x° =$ ☐

b) Find the value of $x°$.

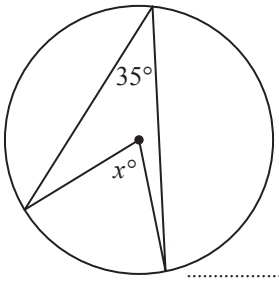


$x° =$ ☐

continued from page 333

Skill 27.9 Finding the value of an angle in a circle (2).

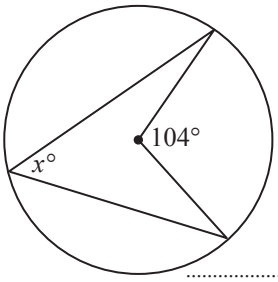
c) Find the value of $x°$.



$x° = 2 \times 35°$

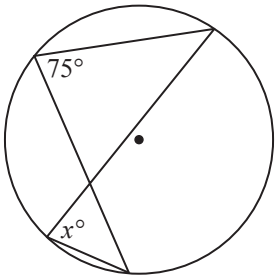
$x° =$ ☐

d) Find the value of $x°$.



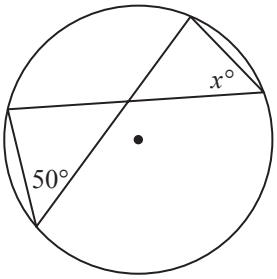
$x° =$ ☐

e) Find the value of $x°$.



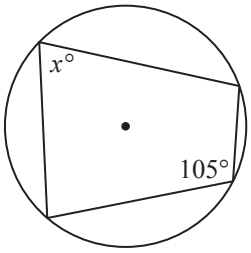
$x° =$ ☐

f) Find the value of $x°$.



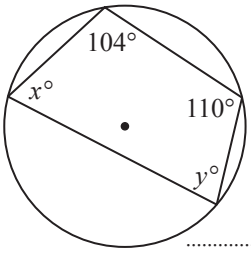
$x° =$ ☐

g) Find the value of $x°$.



$x° =$ ☐

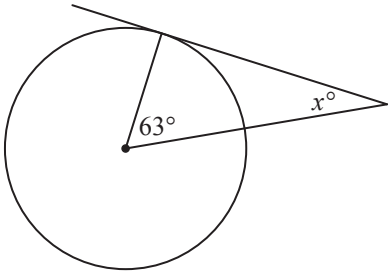
h) Find the values of $x°$ and $y°$.



$x° =$ ☐

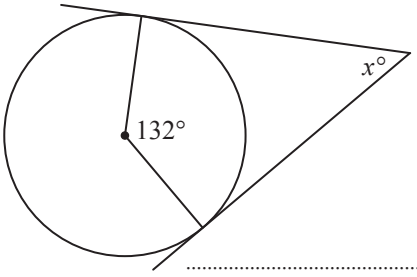
$y° =$ ☐

i) Find the value of $x°$.



$x° =$ ☐

j) Find the value of $x°$.



$x° =$ ☐