13. [Exploring Number]

Skill 13.1 Using 'order of operations' involving a mix of ( ), ×, ÷, + or –

- Simplify inside the brackets.
- Multiply (×) and/or divide (÷) in order from left to right.
- Add (+) and/or subtract (–) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$
subtract inside the brackets
$= 18 \div 6 + 2$
division before addition
$= 3 + 2$
$= \mathbf{5}$

a) $6 + 12 \div 4 \times 3 =$

$= 6 + 3 \times 3$

$= 6 + 9$ $=$ ☐

b) $6 \times 15 - 8 \times 3 =$

$=$

$=$ $=$ ☐

c) $5 + 12 \div 6 \times 3 =$

$=$

$=$ $=$ ☐

d) $3 \times (5 - 3) \times 8 =$

$=$

$=$ $=$ ☐

e) $(15 + 8) - (7 + 6) =$

$=$

$=$ $=$ ☐

f) $120 \div 5 - 6 \times 3 =$

$=$

$=$ $=$ ☐

g) $22 - 8 - (11 - 4) =$

$=$

$=$ $=$ ☐

h) $20 - (15 - 9) + 6 =$

$=$

$=$ $=$ ☐

i) $6 \times (14 + 7) =$

$=$

$=$ $=$ ☐

j) $8 \times 5 \div (7 - 3) =$

$=$

$=$ $=$ ☐

k) $4 + (9 - 4) \times 7 =$

$=$

$=$ $=$ ☐

l) $36 - 2 \times (12 + 5) =$

$=$

$=$ $=$ ☐

m) $144 \div 6 - 4 \times 5 + 18 \div 3 =$

$=$

$=$ $=$ ☐

n) $40 - 3 \times (13 + 5) \div 3 + 12 =$

$=$

$=$ $=$ ☐

Skill 13.2 Using 'order of operations' involving powers and ( ), ×, ÷, + or –

- Simplify inside the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (–) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$ *multiply within brackets first*

$= (6 + 10)^2$ *add inside the brackets*

$= 16^2$

$= \mathbf{256}$

a) $(3 \times 5)^2 =$

$= 15^2$ ..... $=$ ☐

b) $(2 \times 7)^2 =$

$=$ ..... $=$ ☐

c) $(5 + 5 \times 3)^2 =$

$=$ .....

$=$ ..... $=$ ☐

d) $(2 \times 4 + 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

e) $(2 + 8)^2 \div 4 =$

$=$ .....

$=$ ..... $=$ ☐

f) $(7 + 5)^2 \div 8 =$

$=$ .....

$=$ ..... $=$ ☐

g) $5 + (12 - 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

h) $8 + (13 - 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

i) $(4 \times 2 + 2)^2 =$

$=$ .....

$=$ ..... $=$ ☐

j) $(3 \times 4 + 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

k) $3 + (1 + 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

l) $6 + (7 + 1)^2 =$

$=$ .....

$=$ ..... $=$ ☐

m) $(10 - 1)^2 \div (30 - 3) =$

$=$ .....

$=$ ..... $=$ ☐

n) $(10 - 3)^2 \div (12 - 5) =$

$=$ .....

$=$ ..... $=$ ☐

Skill 13.3 Rounding decimal numbers to a given place.

To round terminating decimals to a given place:

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

To round recurring decimals to a given place:

- Write the first 4 digits after the decimal point. (see skill 7.6, page 76)
- Apply the procedure described above for terminating decimals.

Q. Round $0.46\dot{6}$ to 3 decimal places.

A. $0.46\dot{6} = 0.4666...$

$0.466\textcircled{6}..$

$\approx \mathbf{0.467}$

a) Round 0.13 to 1 decimal place.

$0.1\textcircled{3}$ ..... $\approx$ 0.1

b) Round 7.89 to 1 decimal place.

..... $\approx$

c) Round 12.45 to 1 decimal place.

..... $\approx$

d) Round 31.5841 to 2 decimal places.

$31.58\textcircled{4}1$ ..... $\approx$ 31.58

e) Round 24.793 to 2 decimal places.

..... $\approx$

f) Round 4.231 to 2 decimal places.

..... $\approx$

g) Round 3.859 to 1 decimal place.

..... $\approx$

h) Round 50.296 to 2 decimal places.

..... $\approx$

i) Round $4.\dot{7}$ to 2 decimal places.

$4.\dot{7} = 4.77\textcircled{7}7...$ ..... $\approx$ 4.78

j) Round $3.4\dot{2}\dot{2}$ to 2 decimal places.

..... $\approx$

k) Round $0.6\dot{6}$ to 2 decimal places.

..... $\approx$

l) Round $1.7\dot{3}$ to 3 decimal places.

..... $\approx$

m) Round $4.28\dot{8}$ to 3 decimal places.

..... $\approx$

n) Round $0.\dot{1}6\dot{6}$ to 3 decimal places.

..... $\approx$

Skill 13.4 Writing rational approximations of simple irrational numbers.

irrational number $\sqrt{2} \approx 1.41421356$ rational approximation

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the the nearest hundredth.

Q. cos 45° ≈ 0.70711
Write the rational approximation of cos 45° correct to two decimal places.

A. *0.707*11 — circle the third digit
≈ ***0.71*** — 7 ≥ 5, round up by adding 1 to 0

a) $\sqrt{12} \approx 3.46410162$
Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

*3.464*10162 — 4 < 5, round down by keeping 6 ≈ 3.46

b) $\sqrt{20} \approx 4.47213595$
Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

..... ≈ .....

c) $\sqrt{24} \approx 4.89897949$
Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

..... ≈ .....

d) $\sqrt{30} \approx 5.47722558$
Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

..... ≈ .....

e) $\pi \approx 3.14159265$
Write the rational approximation of π correct to three decimal places.

..... ≈ .....

f) $\varphi \approx 1.61803398$ (the golden ratio)
Write the rational approximation of φ correct to three decimal places.

..... ≈ .....

g) sine 15° ≈ 0.25882
Write the rational approximation of sine 15° correct to three decimal places.

..... ≈ .....

h) tan 60° ≈ 1.73205
Write the rational approximation of tan 60° correct to three decimal places.

..... ≈ .....

i) e ≈ 2.71828182 (Euler's number)
Write the rational approximation of e correct to two decimal places.

..... ≈ .....

j) $\sqrt{10} \approx 3.16227766$
Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

..... ≈ .....

Skill 13.5 Writing very large and very small numbers in scientific notation.

243 000	$= 2.43 \times 10^5$	0.000802	$= 8.02 \times 10^{-4}$
basic numeral	**scientific notaion**	**basic numeral**	**scientific notaion**
Very large	Product of: Number ≥ 1 and < 10 Power of 10 with positive index	Very small	Product of: Number ≥ 1 and < 10 Power of 10 with negative index

To find the number greater than or equal to 1 and less than 10:
- Move the decimal point after the first non-zero digit and count the places moved.
- Write this number ignoring the zeros at either end.

To find the power of 10:
- Write base 10 with an index equal to the number of places moved.
- Check the sign of the index: Decimal point moves left ⇒ positive index.
  Decimal point moves right ⇒ negative index.

Q. Write 384 000 000 m, the distance from earth to moon, in scientific notation.

A. $384\,000\,000$ (8 places)
$= 3.84 \times 100\,000\,000$ (8 zeros)
$= \mathbf{3.84 \times 10^8}$ (number ≥ 1 and < 10)

Index = +8 because of the 8 places and large original number (> 3.84)

a) 73 000 written in scientific notation is:
A) 7.3×10^{-4} B) 0.73×10^5 C) 7.3×10^4

$73\,000$ (4 places; 4 zeros; index = +4)
$= 7.3 \times 10\,000 = 7.3 \times 10^4$ = ☐

b) 5 010 000 written in scientific notation is:
A) 5.01×10^6 B) 5.1×10^5 C) 5.01×10^5

$5\,010\,000$
= ..... = ☐

c) Some people have as many as 150 000 hair follicles on their head. Write this in scientific notation.

..... = ☐

d) By 2010 the world's population is estimated to reach 6.84 billion people. Write this in scientific notation.

..... = ☐

e) 0.00304 written in scientific notation is:
A) 3.4×10^{-4} B) 3.04×10^{-3} C) 3.04×10^{-4}

0.00304 (3 places; 3 zeros; index = −3)
$= 3.04 \times 1000 = 3.04 \times 10^{-3}$ = ☐

f) 0.000051 written in scientific notation is:
A) 5.1×10^{-5} B) 5.1×10^5 C) 5.1×10^{-4}

= ..... = ☐

g) Write 0.00000024 m, the diameter of a human hair strand, in scientific notation.

..... = ☐

h) In diamonds the distance between two Carbon atoms is 0.000000142 m. Write this in scientific notation.

..... = ☐

Skill 13.6 Writing a number in scientific notation as a basic numeral.

2.43×10^5	=	$\overbrace{243\,000}$	8.02×10^{-4}	=	$\overbrace{0.000802}$
scientific notation		**basic numeral**	**scientific notation**		**basic numeral**
Product of: Number ≥ 1 and < 10 Power of 10 with positive index		Very large	Product of: Number ≥ 1 and < 10 Power of 10 with negative index		Very small

If the power of 10 is **positive:**
- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 3.1 = 3.1000
  Hint: By convention 37 = 37. = 37.0

If the power of 10 is **negative:**
- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 4.5 = 00004.5
- If the result is less than 1, write a zero in the units place.
  Hint: By convention 0.37 not .37

Q. Write 3.5×10^{-4} m, the diameter of optical fibre, as a basic numeral.

A. 3.5×10^{-4} = (index = −4)
= $\overbrace{00003.5}$ $\times 10^{-4}$ (move decimal point 4 places left; add zeros as place holders)
= **0.00035**

a) 6.2×10^5 is the scientific notation for:
A) 6200 B) 620 000 C) 6.20000

6.2×10^5 = (index = +5)

= $620\,000.$ (5 places right) B

b) 4.12×10^6 is the scientific notation for:
A) 4 120 000 B) 412 000 C) 4.120000

= .....

c) 2.15×10^3 is the scientific notation for:
A) 2.15000 B) 215 000 C) 2150

= .....

d) 1.8×10^7 is the scientific notation for:
A) 1 800 000 B) 18 000 000 C) 180 000

= .....

e) Earth's atmosphere extends upward for 9.65×10^5 m. Write this as a basic numeral.

.....

f) Write 1.4×10^9, China's population in 2010, as a basic numeral.

.....

g) The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
A) 0.0008 B) 8000 C) 0.008

= .....

h) The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
A) 0.00025 B) 0.000025 C) 250 000

= .....

i) Write 2.5×10^{-11} m, the radius of a hydrogen atom, as a decimal number.

.....

j) Write 5×10^{-7} m, the size of a speck of dust, as a decimal number.

.....

Skill 13.7 Using 'order of operations' involving negative numbers.

- Simplify within the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (–) in order from left to right.
- Use the addition, subtraction, multiplication and division rules of negative numbers.

Q. $-48 \div (-8 - 4) + 23 =$

A. $-48 \div (-8 - 4) + 23 =$ *subtract inside the brackets*

$= -48 \div (-12) + 23$ (– ÷ – = +)

$= 4 + 23$ *division before addition*

$= 27$

a) $-6 \times (24 + 6) =$

$= -6 \times 30$ $= -180$

b) $-4 \times (7 + 9) =$

$=$ ..... $=$ ☐

c) $(10 - 2) \div (3 - 7) =$

$=$ ..... $=$ ☐

d) $(11 - 3) \div (1 - 9) =$

$=$ ..... $=$ ☐

e) $(6 - 10) \times (-4 - 8) =$

$=$ ..... $=$ ☐

f) $(-5 - 4) \times (2 - 5) =$

$=$ ..... $=$ ☐

g) $(-9 - 2) \times (12 - 7) =$

$=$ ..... $=$ ☐

h) $(-8 + 3) \times (6 - 12) =$

$=$ ..... $=$ ☐

i) $8 \times 12 \div (2 - 6) =$

$=$ .....

$=$ ..... $=$ ☐

j) $5 \times 14 \div (3 - 10) =$

$=$ .....

$=$ ..... $=$ ☐

k) $-5 - 4 \times (11 - 9) =$

$=$ .....

$=$ ..... $=$ ☐

l) $-7 + 2 \times (15 - 4) =$

$=$ .....

$=$ ..... $=$ ☐

m) $-24 \div (-3 - 3) + 17 =$

$=$ .....

$=$ ..... $=$ ☐

n) $-12 \times 5 - 45 \div 9 =$

$=$ .....

$=$ ..... $=$ ☐

o) $-18 + 4 \times (7 + 8) \div 10 - 2 =$

$=$ .....

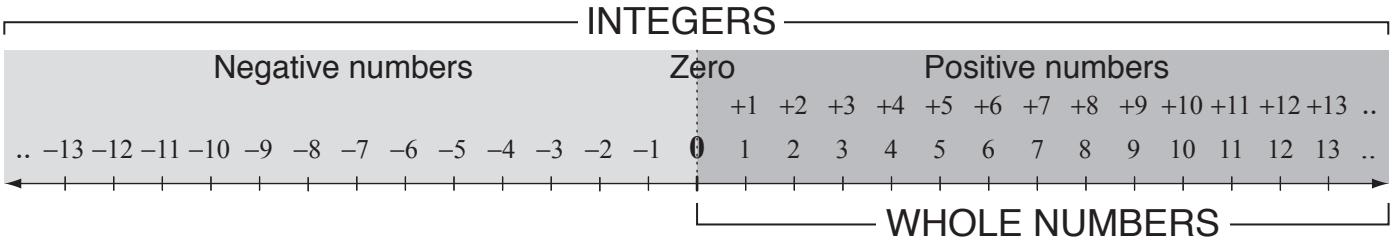
$=$ ..... $=$ ☐

p) $5 - 3 \times (6 + 2) \div 2 - 8 =$

$=$ .....

$=$ ..... $=$ ☐

Skill 13.8 Recognising whole numbers and integers.



- Decide if a number is a whole number or an integer, based on their definition (see Glossary) and hints below.

Hints: *Negative numbers, fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are not whole numbers.*
Any fraction whose numerator is divisible by the denominator is a whole number: $\frac{12}{4} = 3$
Any decimal with only zeros after the decimal point is a whole number: $8.00 = 8$
Fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are not integers.
Any fraction whose numerator is divisible by the denominator is an integer: $\frac{5}{1} = 5, -\frac{12}{4} = -3$
Any decimal with only zeros after the decimal point is an integer: $-8.00 = -8$
Any square root of a perfect square is an integer: $\sqrt{16} = 4$

Q. Choose the whole numbers from this list:
$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

A. *-7 is negative, so not a whole number*
$\frac{8}{2} = 8 \div 2 = 4$ is a whole number
$-\frac{1}{3}$ is a fraction, so not a whole number
-3.6 is a decimal, so not a whole number
So $\frac{8}{2}$***, 0, 50*** *are whole numbers.*

a) Choose the whole numbers from this list:
$7.43, (89), -5, 3\frac{1}{5}, (14), 0.6$

b) Choose the whole numbers from this list:
$567, 0.73, -4, \frac{3}{10}, 12, 0$

c) Choose the whole numbers from this list:
$1.4142, 18, -5.\overline{9}, \frac{4}{11}, -5, 143$

d) Choose the whole numbers from this list:
$-25, 0.6666..., 34, \frac{5}{7}, -1, 8.93567, 2$

e) Choose the integers from this list:
$-3.5, 11, 2.\overline{14}, -1, 3\frac{2}{7}, 2$

f) Choose the integers from this list:
$3.14, \frac{16}{4}, -3, -0.\overline{72}, \sqrt{25}$

g) Choose the integers from this list:
$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

h) Choose the integers from this list:
$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

Skill 13.9 Recognising rational and irrational numbers.

A number is **rational** if:

- It can be written as a fraction of 2 integers.

Hint: All integers are rational numbers.

$-2,\ 700,\ \sqrt{16},\ \frac{5}{1},\ \frac{25}{5}$

All terminating decimals are rational.

$2.16,\ -5.753469$

All recurring decimals are rational.

$0.57575757... = 0.\dot{5}\dot{7}$

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-recurring digits after the decimal point.

Example: 2.52849302953...

Hint: Square roots of prime numbers and rational numbers that are not perfect squares are irrational numbers.

$\sqrt{5},\ \sqrt{18}$

Q. Which numbers are rational?

A) $-\sqrt{\frac{3}{5}}$

B) 0.999...

C) π

D) $\frac{11}{2}$

A. *$-\sqrt{\frac{3}{5}}$ is irrational, because $\frac{3}{5}$ is not a perfect square.*

0.999... is rational, because it is a recurring decimal.

π is irrational, because it has infinite non-recurring digits after the decimal point ($\pi \approx 3.1415926535....$)

$\frac{11}{2}$ is rational, because it is a fraction.

*So **B and D** are rational.*

a) Is $\sqrt{7}$ a rational or an irrational number?

irrational

b) Is $4.12\dot{6}\dot{3}$ a rational or an irrational number?

c) Is $\frac{48}{25}$ a rational or irrational number?

d) Is 1.72430982... a rational or irrational number?

e) Is –60 a whole number, an integer or an irrational number?

f) Is 2.676767... a whole number, an integer or a rational number?

g) Is $-\frac{7}{2}$ a whole number, an integer or a rational number?

h) Is –12 000 a whole number, an integer or an irrational number?

i) Which is **not** a rational number?

A) –6 B) 0.18952

C) π D) $-\sqrt{9}$

j) Which is **not** a rational number?

A) $-0.3\dot{1}$ B) $\sqrt{3}$

C) 2.135135135... D) $\frac{11}{49}$

k) Which is an irrational number?

A) 3 B) –2.5

C) $\sqrt{4}$ D) $-\sqrt{2}$

l) Which is an irrational number?

A) $2.\dot{6}$ B) 6.15

C) $\sqrt{7}$ D) $5\frac{3}{10}$

m) Which numbers are rational?

A) $\sqrt{8}$ B) $6.5\dot{9}$

C) –4.131133111333... D) 3.161616...

n) Choose the rational numbers from this list:

$-2012,\ \pi,\ 0,\ \frac{28}{11},\ -\sqrt{6}$

Skill 13.10 Recognising classes of numbers.

$\mathbb{R}$ REAL NUMBERS

IRRATIONAL
π, φ, e, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,
2.6293045632....
cos 30°

$\mathbb{Q}$
RATIONAL
$-2\frac{3}{7}$, 3.010101..., $\frac{4}{10}$, 0.56, $\sqrt{\frac{4}{9}}$

$\mathbb{Z}$
Integers
..., –3, –2, –1, 0, 1, 2, 3, ...

$\mathbb{N}$
Natural (Whole Numbers)
0, 1, 2, 3, 4, 5, 6, ...

Hint: Rational numbers include integers, terminating decimals and recurring decimals.
Irrational numbers include infinite non-recurring decimals.
$\mathbb{N}$ included in $\mathbb{Z}$, $\mathbb{Z}$ included in $\mathbb{Q}$, $\mathbb{Q}$ included in $\mathbb{R}$
Irrational number included in $\mathbb{R}$

Q. Which classification describes $-\sqrt{81}$?
A) integer and irrational
B) rational and real
C) irrational and rational
D) real and natural

A. $-\sqrt{81} = -9$

integer	✔
rational	✔
real number	✔
natural	✘
irrational	✘

*So **B** is the correct description.*

a) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
4.327	false	true	false	true

b) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
–500				

c) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
π				

d) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{3}{14}$				

e) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\sqrt{26}$				

f) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{36}{9}$				

g) Which classification describes 0.65291...?
A) integer and rational
B) rational and real
C) integer and irrational
D) irrational and real

h) Which classification describes $-\sqrt{49}$?
A) integer and rational
B) irrational and real
C) integer and irrational
D) rational and irrational

i) Which classification describes $0.\dot{1}53846\dot{}$?
A) integer and irrational
B) irrational and real
C) integer and rational
D) rational and real

j) Which classification describes $\frac{257}{43}$?
A) integer and rational
B) irrational and real
C) rational and real
D) rational and irrational

Skill 13.11 Comparing and ordering rational and irrational numbers.

- Express the numbers as decimals.
- Round the decimal numbers to one or two decimal places, as needed in the question.
  Examples: $\pi = 3.1415926... \approx 3.14$ correct to 2 decimal places
  $\frac{5}{3} = 1.6666... \approx 1.66$ correct to 2 decimal places
  $\sqrt{3} = 1.732050808... \approx 1.73$ correct to 2 decimal places
- Find the perfect squares greater than (>) and less than (<) the number under the square root.
- Compare and order the terminating decimals.

Q. Place in ascending order:
$\sqrt{10}, \frac{10}{3}, 3.21, \pi, \sqrt{12}$

A. *express numbers as decimals*

$\sqrt{10} = 3.16227766... \approx 3.16$

$\frac{10}{3} = 3.33333... \approx 3.33$ *round to 2 decimal places*

$\pi = 3.1415... \approx 3.14$

$\sqrt{12} = 3.46410161... \approx 3.46$

$\Rightarrow \quad 3.14 < 3.16 < 3.21 < 3.33 < 3.46$

OR $\pi < \sqrt{10} < 3.21 < \frac{10}{3} < \sqrt{12}$

The answer is $\boldsymbol{\pi, \sqrt{10}, 3.21, \frac{10}{3}, \sqrt{12}}$

a) Which number is greater?
$\sqrt{21}$ or 5

$16 < 21 < 25 \Rightarrow 4 < \sqrt{21} < 5 \Rightarrow$ 5

b) Which number is greater?
$\sqrt{72}$ or 8

..... $\Rightarrow$ ☐

c) Which number is smaller?
1.41 or $\sqrt{2}$

..... $\Rightarrow$ ☐

d) Which number is greater?
$\sqrt{8}$ or 3

..... $\Rightarrow$ ☐

e) Which number is greater?
π or $\sqrt{9}$

..... $\Rightarrow$ ☐

f) Which number is smaller?
$\frac{5}{2}$ or $\sqrt{6}$

..... $\Rightarrow$ ☐

g) Which number is greater?
$\sqrt{18}$ or 4

..... $\Rightarrow$ ☐

h) Which number is smaller?
$\sqrt{25}$ or $5.\dot{1}$

..... $\Rightarrow$ ☐

i) Place in ascending order:
$\sqrt{5}, \frac{7}{3}, 2.2\dot{2}, \frac{5}{2}, 2.4$

.....

..... ☐

j) Place in ascending order:
$\sqrt{8}, \frac{8}{3}, \frac{10}{4}, 2.76, \sqrt{7}$

.....

..... ☐