19. [Factorisation]

Skill 19.1 Factorising by finding the HCF of the coefficients.

MM5.2 **1 1** 2 2 3 3 4 4
MM10 **1** 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the HCF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise $15a - 24$

A. $15a - 24$

$3 \times 5 = 15$ and $3 \times 8 = 24$

HCF of 15 and 24 is 3

Write the HCF before the ( ) — Remaining factors are $5a$ and 8

$= 3(5a - 8)$ — Keep the sign

a) Factorise $4k - 16$ = $4(k - 4)$

$4 \times k = 4k$ and $4 \times 4 = 16 \Rightarrow$ HCF is 4

b) Factorise $4x + 8$ =

c) Factorise $6s + 18$ =

d) Factorise $3u - 15$ =

e) Factorise $9m - 2$ =

f) Factorise $14n + 21$ =

g) Factorise $2y + 10z$ =

h) Factorise $4a - 12b$ =

i) Factorise $6d + 14e$ =

j) Factorise $16uv - 4$ =

k) Factorise $12k - 8l$ =

l) Factorise $4g + 4h - 6$ =

m) Factorise $3m - 6n + 9$ =

n) Factorise $10v - 5w + 15$ =

o) Factorise $5h^2 - 10i + 25j$ =

p) Factorise $6r^2 - 27s + 9t$ =

Skill 19.2 Factorising by finding the HCF of coefficients and pronumerals.

MM5.2 1 1 2 2 3 3 4 4
MM10 1 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $18kl - 24k$

A. $18kl - 24k$

$6 \times 3 = 18$ and $6 \times 4 = 24$

HCF of 18 and 24 is 6

k is common to both terms CF is k

Remaining factors are $3l$ and 4

Write all CF's before the ( ) $= 6k(3l - 4)$ Keep the sign

a) Factorise $ab + 5b$

ab = ba

b is common to both terms $= b(a + 5)$

b) Factorise $de + d$

..... $=$ .....

c) Factorise $7e + ef$

..... $=$ .....

d) Factorise $3st + 4s$

..... $=$ .....

e) Factorise $8ab - 4b$

..... $=$ .....

f) Factorise $15g + 20gh$

..... $=$ .....

g) Factorise $wx - xy$

..... $=$ .....

h) Factorise $2jk + 2kl$

..... $=$ .....

i) Factorise $uv - 3vw$

..... $=$ .....

j) Factorise $8ab + 4bc$

..... $=$ .....

k) Factorise $12qr + 8rs$

..... $=$ .....

l) Factorise $15de - 6ef$

..... $=$ .....

m) Factorise $10cd - 8d$

..... $=$ .....

n) Factorise $15m - 10mn$

..... $=$ .....

o) Factorise $21qr + 14pq$

..... $=$ .....

p) Factorise $6tu + 18uv$

..... $=$ .....

q) Factorise $6xy + 9yz$

..... $=$ .....

r) Factorise $10gh - 25gi$

..... $=$ .....

Skill 19.3 Factorising to simplify expressions involving large numbers.

- Find the number repeating in both products.
- Write this number in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise and evaluate
$45 \times 7 + 45 \times 3$

A. $45 \times 7 + 45 \times 3$ (45 is repeating)
$= 45 \times (7 + 3)$
$= 45 \times 10$
$= \mathbf{450}$

a) Factorise and evaluate
$99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$
$= 99 \times 1$ $= 99$

b) Factorise and evaluate
$15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$
$=$ $=$

c) Factorise and evaluate
$987 \times 2 + 987 \times 8$

$=$
$=$ $=$

d) Factorise and evaluate
$40 \times 8 + 40 \times 12$

$=$
$=$ $=$

e) Factorise and evaluate
$23 \times 37 + 23 \times 63$

$=$
$=$ $=$

f) Factorise and evaluate
$25 \times 26 + 25 \times 24$

$=$
$=$ $=$

g) Factorise and evaluate
$999 \times 9 - 999 \times 8$

$=$
$=$ $=$

h) Factorise and evaluate
$87 \times 19 - 87 \times 9$

$=$
$=$ $=$

i) Factorise and evaluate
$-4 \times 14 - 4 \times 6$

(Both terms are negative so CF is negative; − + = −)

$= -4 \times (14 + 6)$
$=$ $=$

j) Factorise and evaluate
$-9 \times 33 - 9 \times 67$

$=$
$=$ $=$

Skill 19.4 Factorising involving squared terms.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $2wx - 12w^2x$

A. $2wx - 12w^2x$

$2 \times 1 = 2$ and $2 \times 6 = 12$

HCF of 2 and 12 is 2

wx is common to both terms CF is wx

Remaining factors are 1 and $6w$

Write all CF's before the ( ) → $= 2wx(1 - 6w)$ ← Keep the sign

a) Factorise $2j^2k + 5j$

$CF = j$ ..... $= j(2jk + 5)$

Write all CF's outside the ( )

b) Factorise $e^2 + 7e$

..... =

c) Factorise $h + 4h^2$

..... =

d) Factorise $m^2 - 9m$

..... =

e) Factorise $3c - 12c^2$

..... =

f) Factorise $4f^2 + 6f$

..... =

g) Factorise $fg^2 + f$

..... =

h) Factorise $10b - 16ab^2$

..... =

i) Factorise $p^2q - 3p$

..... =

j) Factorise $12i - 18hi^2$

..... =

k) Factorise $14bc + 2b^2c$

..... =

l) Factorise $5r^2s - r^2t$

..... =

m) Factorise $vw + 7v^2 - 3vwx$

..... =

n) Factorise $8j^2 - 24jk + 12jl$

..... =

o) Factorise $f^3g^2 + fg^2$

..... =

p) Factorise $p^3q^2 + p^2q + pq$

..... =

Skill 19.5 Factorising negative terms.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the HCF and any other CF in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $-10r^2 - 5r$

A. $-10r^2 - 5r$
$5 \times 2 = 10$ and $5 \times 1 = 5$
HCF of 10 and 5 is 5
r is common to both terms CF is r
$-$ is common to both terms CF is $-$
Remaining factors are r and 1
Write all CF before the ( ) $= -5r(r + 1)$

"$-$" is common to both terms

a) Factorise $-7a - 21$ $= -7(a + 3)$

$7 \times 1 = 7$ and $7 \times 3 = 21 \Rightarrow$ HCF is 7

b) Factorise $-4k - 12$ $=$ .....

$4 \times 1 = 4$ and $4 \times 3 = 12 \Rightarrow$ HCF is

c) Factorise $-6g - 15$ $=$ .....

d) Factorise $-6e - 14$ $=$ .....

e) Factorise $-2h^2 - 6h$ $=$ .....

f) Factorise $-8z^2 - 28z$ $=$ .....

g) Factorise $-12i^3 - 9ij$ $=$ .....

h) Factorise $-t^3 - 5t^2u$ $=$ .....

i) Factorise $-6bc^2 + 3c^2 =$ .....

j) Factorise $-5x^2y^2 - xy^3 =$ .....

k) Factorise $-2x^3 - 4xy$ $=$ .....

l) Factorise $-4m^3 - 12mn^2 + 18m =$ .....

m) Factorise $-2k^3 + 6k^3l + 8k$ $=$ .....

n) Factorise $-2hi^3 + 3h^2i - 5h^2$ $=$ .....

Skill 19.6 Factorising by finding binomial factors.

- Find any common factors (CF).
  Hint: It might help to think of common factors that are expressions like (d + 2) as a blob ■.
- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $2(r-1)-r(r-1)$

A. $2(r-1)-r(r-1)$ — Consider $(r-1) = ■$
$= 2■ - r■$ — Keep the sign
$= ■(2-r)$
$= \mathbf{(r-1)(2-r)}$

a) Factorise
$d(d+2)+8(d+2)$ — Consider $(d+2) = ■$
$= d■ + 8■$ — Keep the sign
$= ■(d+8)$ $= \boxed{(d+2)(d+8)}$

b) Factorise
$2(h-3)+h(h-3)$
$= 2■ + h■$
$=$ ..... $=$ ☐

c) Factorise
$5(x+4)+x(x+4)$
$=$ .....
$=$ ..... $=$ ☐

d) Factorise
$b(b-7)+6(b-7)$
$=$ .....
$=$ ..... $=$ ☐

e) Factorise
$a(a+2)-9(a+2)$
$=$ .....
$=$ ..... $=$ ☐

f) Factorise
$z(z-5)-(z-5)$
$=$ .....
$=$ ..... $=$ ☐

g) Factorise
$j(j+4)+j+4$
$=$ .....
$=$ ..... $=$ ☐

h) Factorise
$m(n-2)+4(n-2)$
$=$ .....
$=$ ..... $=$ ☐

i) Factorise
$3x(2x-5)-4(2x-5)$
$=$ .....
$=$ ..... $=$ ☐

j) Factorise
$d(c+5)-(c+5)$
$=$ .....
$=$ ..... $=$ ☐

k) Factorise
$q(s-3)+t(s-3)$
$=$ .....
$=$ ..... $=$ ☐

l) Factorise
$6v(2w-1)+4(2w-1)$
$=$ .....
$=$ ..... $=$ ☐

Skill 19.7 Factorising four terms by grouping 2 and 2.

- Begin factorising by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factorise again by finding the common binomial factor. (see skill 19.6, page 194)
  Hint: It might help to think of binomial factors that are expressions like (d + 2) as a blob ■.
- Take out the binomial factor or blob and write it in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

Q. Factorise $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$ — Keep the sign

$= m(m + 3) + 5(m + 3)$

$= m(■) + 5(■)$ — Consider $(m + 3) = ■$

$= ■(m + 5)$

$= \mathbf{(m + 3)(m + 5)}$

a) Factorise

$c^2 + 8c + 3c + 24$ — Group 2 and 2, Factorise each group

$= c(c + 8) + 3(c + 8)$ — Factorise again, Consider $(c + 8) = ■$

$= ■(c + 3)$ $= (c + 8)(c + 3)$

b) Factorise

$a^2 + 3a + 2a + 6$

$= a(a + 3) + 2(a + 3)$

$=$ $=$

c) Factorise

$s^2 + 6s + 5s + 30$

$=$

$=$ $=$

d) Factorise

$h^2 + 5h + 4h + 20$

$=$

$=$ $=$

e) Factorise

$v^2 + 7v + 3v + 21$

$=$

$=$ $=$

f) Factorise

$4n + n^2 + 16 + 4n$

$=$

$=$ $=$

g) Factorise

$6t + t^2 - 42 - 7t$

$=$

$=$ $=$

h) Factorise

$4b + 4 - b^2 - b$

$=$

$=$ $=$

i) Factorise

$5p - 10 + p^2 - 2p$

$=$

$=$ $=$

j) Factorise

$q^2 + 5q - 4q - 20$

$=$

$=$ $=$

Skill 19.8 Factorising using the difference of perfect squares.

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$(a + b)(a - b) = a \times a + a \times -b + b \times a + b \times -b$

$= a^2 - ab + ba - b^2$

① ② ③ ④

$= a^2 - b^2$

Group like terms

Q. Factorise $5w^2 - 20$

A. $5w^2 - 20$

Take out the CF of 5

$= 5(w^2 - 4)$

$4 = 2^2$

$= 5(w^2 - 2^2)$

Use $a^2 - b^2 = (a + b)(a - b)$ where $a = w$ and $b = 2$

$= \mathbf{5(w + 2)(w - 2)}$

a) Factorise $c^2 - 81$

$b^2 = 81$, $b = 9$

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2$ $= (c + 9)(c - 9)$

b) Factorise $y^2 - 4$

.....

.....

c) Factorise $d^2 - e^2$

.....

.....

d) Factorise $36 - h^2$

.....

.....

e) Factorise $4j^2 - 9$

.....

.....

f) Factorise $2c^2 - 50$

.....

.....

g) Factorise $p^2 - 81q^2$

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h) Factorise $80 - 5y^2$

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.....

i) Factorise $9a^2 - 36b^2$

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.....

j) Factorise $75 - 3z^2$

.....

.....

k) Factorise $3d^2 - 27$

.....

.....

l) Factorise $100 - 4k^2$

.....

.....

Skill 19.9 Factorising quadratic trinomials.

- Write two sets of brackets. Because x^2 can only be produced from $x \times x$, write the factors of the squared pronumeral in the brackets $(x \quad)(x \quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factorise $x^2 - 9x + 8$

A. $x^2 - 9x + 8$

$= (x \quad)(x \quad)$ — Write x in the brackets

$8 = 1 \times 8 = -1 \times -8 = 2 \times 4 = -2 \times -4$ — List the factors of +8

Only $-1x$ and $-8x$ can make $-9x$ — Determine the x terms

$= x^2 - 1x - 8x + 8$

$= \mathbf{(x - 1)(x - 8)}$

CHECK — Expand the brackets

$= (x - 1)(x - 8)$

$= x^2 - 1x - 8x + 8$

$= x^2 - 9x + 8 = \mathbf{(x - 1)(x - 8)}$ ✔

AND/OR consider

$= x^2 - 1x - 8x + 8$ — Group 2 and 2

$= x(x - 1) - 8(x - 1)$ — Factorise each group

$= x(\blacksquare) - 8(\blacksquare)$

$= \blacksquare(x - 8)$ — Factorise again. Consider $(x - 1) = \blacksquare$

$= \mathbf{(x - 1)(x - 8)}$

a) Find the missing factor
$x^2 + 7x + 10$

$10 = 2 \times 5 = -2 \times -5$

$5x + 2x = 7x$ $= (x + 5)(x + 2)$

b) Find the missing factor
$d^2 - 4d + 4$

$4 = 2 \times 2 = -2 \times -2$

$= (d - 2)(\quad)$

c) Find the missing factor
$s^2 + 4s + 3$

$= (s + 3)(\quad)$

d) Find the missing factor
$g^2 + 8g + 15$

$= (g + 5)(\quad)$

e) Factorise
$m^2 + 2m - 24$

Which pair can be added to get +2m?

$-24 = -4 \times 6 = 4 \times -6$

$6m - 4m = 2m$ $=$ ☐

f) Factorise
$j^2 + 11j + 24$

$=$ ☐

g) Factorise
$y^2 + 5y + 4$

$=$ ☐

h) Factorise
$z^2 - 6z + 8$

$=$ ☐

i) Factorise
$c^2 - 6c + 5$

$=$ ☐

j) Factorise
$p^2 - 6p - 16$

$=$ ☐