

13. [Integers]

continues on page 92

Skill 13.1 Comparing and ordering integers (1).

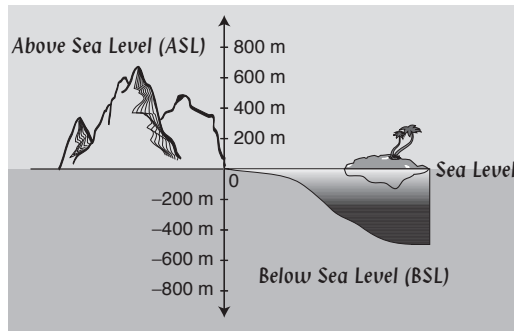
MMBlue 11 22 33 44
MMGreen 11 22 33 44

- Use a number line.

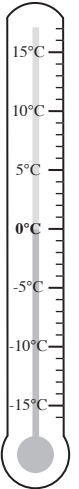
*Hint: Numbers decrease as you move to the left or down and increase as you move to the right or up.
A negative number is always smaller than a positive number.*



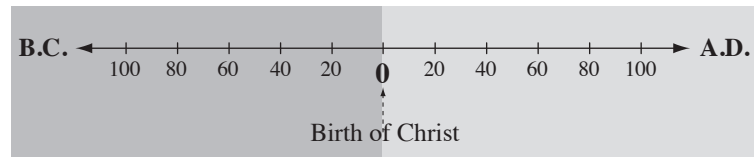
An altitude is lower when further down, below sea level (BSL) and higher when further up, above sea level (ASL).



Temperatures below zero are lower than temperatures above zero.



Years before Christ (B.C.) are prior to his birth at 0 years.
Years after this are A.D. (anno domini).



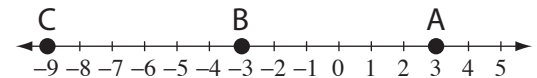
Q. Who won the 2008 Women's Open Golf Tournament?

[In golf the lowest score wins.]

- A) +3 A. Sorenstam
- B) -3 P. Creamer
- C) -9 I. Park

A. C

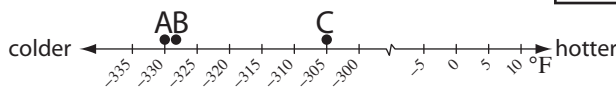
Find the lowest score to determine the winner.



a) Which of Saturn's moons has the highest temperature?

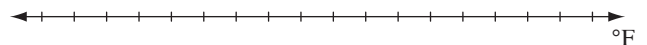
- A) -330°F Enceladus
- B) -328°F Mimas
- C) -305°F Tethys

C



b) Which temperature for oxygen is higher?

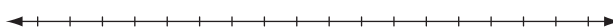
- A) -297°F boiling point
- B) -362°F melting point



c) Who won the 2008 British Open Golf Tournament?

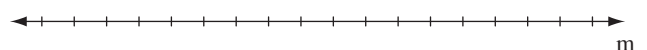
[In golf the lowest score wins.]

- A) -7 P. Harrington
- B) +14 S. Lyle
- C) -2 M. Weir



d) Which body of water is at the lowest altitude?

- A) -28 m Caspian Sea
- B) -408 m Dead Sea
- C) -15 m Lake Eyre

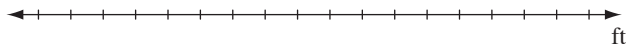


Skill 13.1 Comparing and ordering integers (2).

MMBlue 1 1 2 2 3 3 4 4
MMGreen 1 1 2 2 3 3 4 4

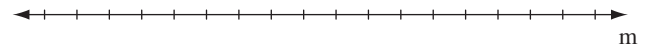
e) Which location has the lowest altitude?

- A) 10 ft above sea level
Amsterdam (Netherlands)
- B) 436 ft below sea level
Qattara Depression (Egypt)
- C) 7970 ft above sea level
Machu Picchu (Peru)



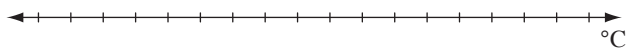
f) Which location has the highest altitude?

- A) 10 m below sea level
Laguna Salada (Mexico)
- B) 7 m below sea level
Lammefjord (Denmark)
- C) 19 m above sea level
Vatican City (Italy)



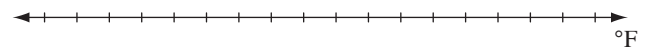
g) Which location recorded the lowest temperature?

- A) -25.6°C Kabul
- B) $+14.1^{\circ}\text{C}$ Christmas Island
- C) -15.2°C La Paz



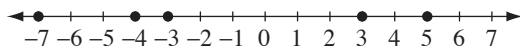
h) Which continent has the lowest recorded temperature?

- A) -81°F North America
- B) -9°F Australia
- C) -67°F Europe



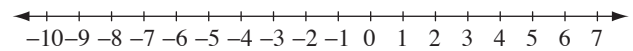
i) Arrange in order from smallest to largest:

$-4, -7, 5, -3, 3$



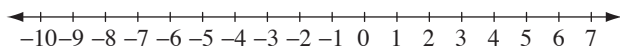
j) Arrange in order from largest to smallest:

$0, 8, -9, 6, -4$



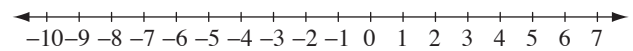
k) Arrange in order from largest to smallest:

$-10, 8, 1, -8, 4$



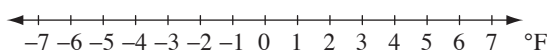
l) Arrange in order from smallest to largest:

$-2, -6, 0, -3, 5$



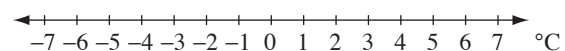
m) Arrange in order from coldest to warmest:

$2^{\circ}\text{F}, -3^{\circ}\text{F}, 4^{\circ}\text{F}, -5^{\circ}\text{F}$



n) Arrange in order from warmest to coldest:

$-1^{\circ}\text{C}, -5^{\circ}\text{C}, 5^{\circ}\text{C}, -3^{\circ}\text{C}$



Skill 13.2 Comparing integers using 'less than' and 'greater than'.

MMBlue 1 1 2 2 3 3 4 4
MMGreen 1 1 2 2 3 3 4 4

- Use a number line.

Hint: A negative number is always smaller than a positive number.

The larger the negative number the lesser the value, e.g. -9 is less than ($<$) -2

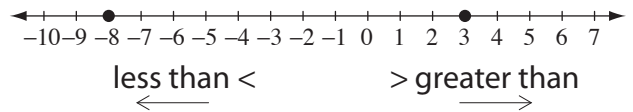
The smaller the negative number the greater the value, e.g. -4 is greater than ($>$) -6

Q. Use $<$ or $>$ to make a true statement.

$$3 \quad \square \quad -8$$

A. $3 > -8$

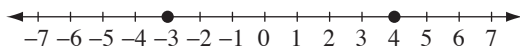
3 is greater than -8



a) Use $<$ or $>$ to make a true statement.

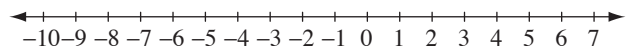
a negative number is less than a positive number

$$-3 \quad \square \quad 4$$



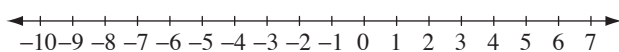
b) Use $<$ or $>$ to make a true statement.

$$-5 \quad \square \quad 0$$



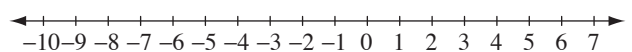
c) Use $<$ or $>$ to make a true statement.

$$-4 \quad \square \quad -9$$



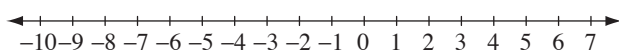
d) Use $<$ or $>$ to make a true statement.

$$-6 \quad \square \quad 3$$



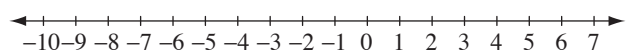
e) Use $<$ or $>$ to make a true statement.

$$2 \quad \square \quad -1$$



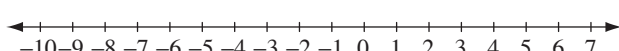
f) Use $<$ or $>$ to make a true statement.

$$-3 \quad \square \quad -7$$



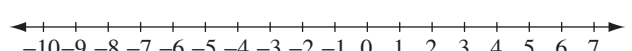
g) Use $<$ or $>$ to make a true statement.

$$-9 \quad \square \quad 0$$



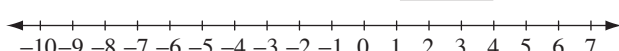
h) Use $<$ or $>$ to make a true statement.

$$3 \quad \square \quad -5$$



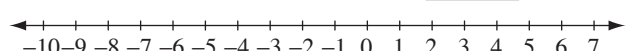
i) Use $<$ or $>$ to make a true statement.

$$4 \quad \square \quad -7$$



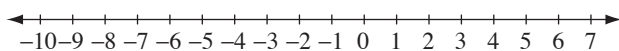
j) Use $<$ or $>$ to make a true statement.

$$-4 \quad \square \quad -2$$



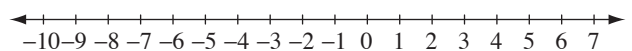
k) Use $<$ or $>$ to make a true statement.

$$-8 \quad \square \quad -5$$



l) Use $<$ or $>$ to make a true statement.

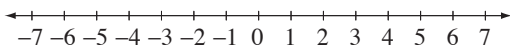
$$-2 \quad \square \quad -4$$



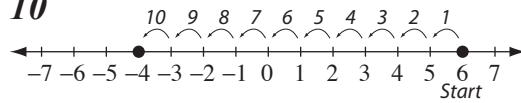
Skill 13.3 Modeling integer subtraction on a number line.

- Determine the value of each mark on the number line.
 - Count the number of spaces between the integers using the number line.
- Hint: Use short cuts such as: counting to zero, counting by tens.*

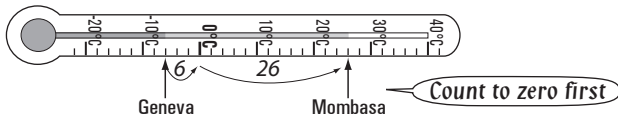
Q. How many units between 6 and -4 ?



A. 10

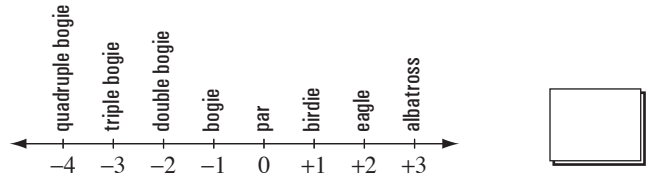


a) How much cooler is it in Geneva than Mombasa?

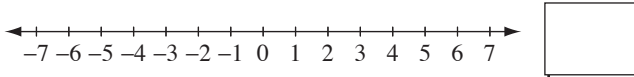


$6 + 26 = \boxed{32 \text{ } ^\circ\text{C}}$

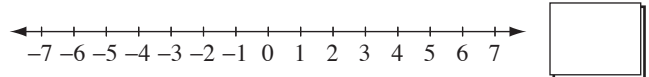
b) If Karrie Webb scores a triple bogie and Greg Norman scores an eagle, what is the difference between their scores?



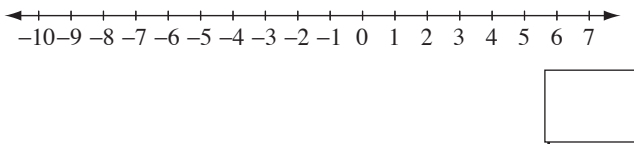
c) How many units between 5 and -4 ?



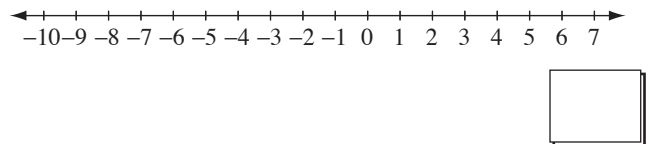
d) How many units between -5 and 3?



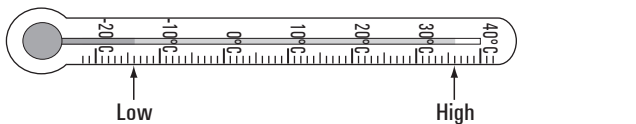
e) How many units between -9 and 2?



f) How many units between 6 and -7 ?

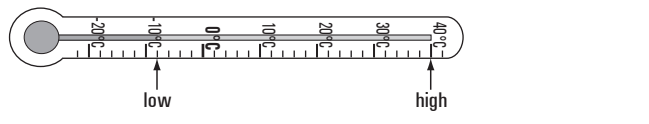


g) What is the difference between the highest and the lowest temperatures recorded in Dunedin, New Zealand?



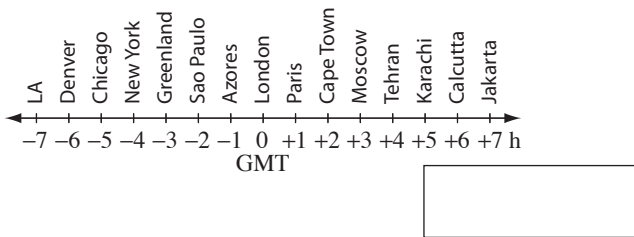
$\dots = \boxed{\text{ } ^\circ\text{C}}$

h) What is the difference between the highest and the lowest temperatures recorded in Rome, Italy?

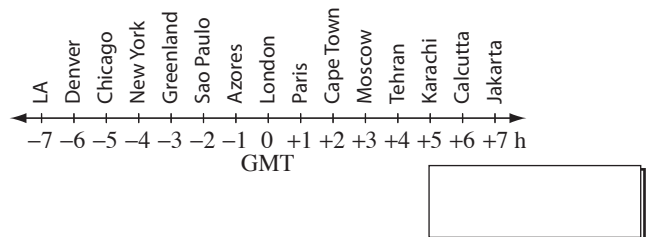


$\dots = \boxed{\text{ } ^\circ\text{C}}$

i) What is the time difference in hours between Denver and Cape Town?



j) What is the time difference in hours between Karachi and New York?

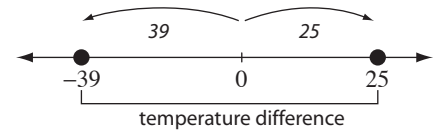


Skill 13.4 Modeling integer subtraction using absolute value.

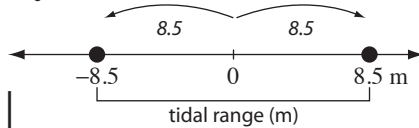
- Visualize the position of the values on a number line.
- Add the absolute values. (see Glossary, page 317)

Q. In Vienna (Austria) the highest recorded temperature is 25°C and the lowest is -39°C . What is the temperature difference?

$$\begin{aligned} \text{A. } & |25| + |-39| \\ &= 25 + 39 \\ &= \mathbf{64^{\circ}\text{C}} \end{aligned}$$



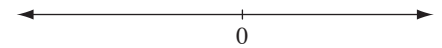
a) The Bay of Fundy, Canada has a high tide of 8.5 m and a low tide of -8.5 m. What is the tidal range for the Bay of Fundy?



$$|8.5| + |-8.5|$$

$$8.5 + 8.5 = \boxed{17 \text{ m}}$$

b) The lowest point in Japan is Lake Hachirogata at -4 m and the highest point is Mt Fujiyama at 3776 m. What is the height difference?



$$= \boxed{\text{m}}$$

c) Sparrow Hills station is the highest station in the Russian metro rail system with an altitude of 220 m above sea level. Park Pobedy is the lowest station at 90 m below sea level. What is their height difference?

$$= \boxed{\text{m}}$$

d) In Reykjavik (Iceland) the highest recorded temperature is 26°C and the lowest is -25°C . What is the temperature difference?

$$= \boxed{^{\circ}\text{C}}$$

e) In Luxembourg the highest recorded temperature is 38°C and the lowest is -23°C . What is the temperature difference?

$$= \boxed{^{\circ}\text{C}}$$

f) In Shanghai (China) the highest recorded temperature is 40°C and the lowest is -12°C . What is the temperature difference?

$$= \boxed{^{\circ}\text{C}}$$

g) The lowest point on the African continent is -156 m at Lake Assal and the highest is 5895 m at Mt Kilimanjaro. What is the height difference?

$$= \boxed{\text{m}}$$

h) The highest point in Europe is 18,481 ft at Mt Elbrus and the lowest is -92 ft in the Caspian Sea. What is the height difference in Europe?

$$= \boxed{\text{ft}}$$

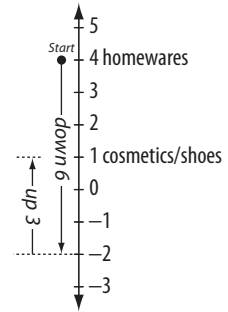
Skill 13.5 Modeling integer addition on a number line.

- Start at the given point on the number line.
- Count up or down the number of spaces as directed.

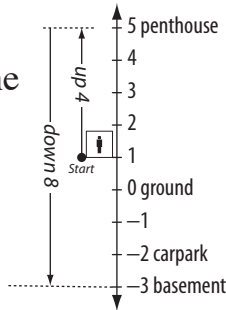
Q. From homewares Marion rides the elevator down 6 levels and up 3 levels. At what level is Marion now?



A. down 6 levels (add -6)
up 3 levels (add $+3$)
 \Rightarrow **cosmetics/shoes**



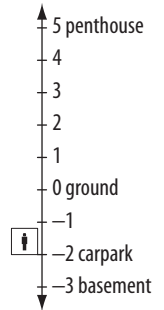
a) From level 1 Hutch rides the elevator up 4 levels and down 8. At what level is Hutch now?



up 4 levels (add $+4$)

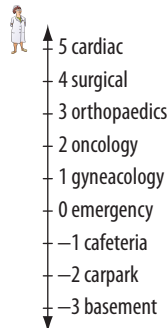
down 8 levels (add -8) \Rightarrow **basement**

b) From the carpark Kwong rides the elevator down 1 level and up 3 levels. At what level is Kwong now?



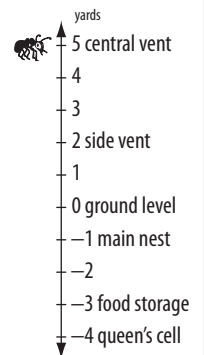
\Rightarrow

c) A nurse starts in the cardiac ward, goes down 6 levels and then up 3 levels. Where does she finish?



\Rightarrow

d) A termite entered his tower via the central vent, got food and went to the main nest. How far did the termite travel?



\Rightarrow

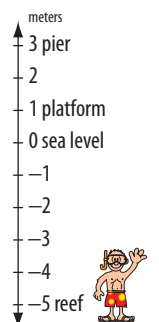
yd

e) From carpark (A) Todd rides the elevator down 2 levels and up 7 levels. At what level is Todd now?



\Rightarrow

f) A snorkeler at the reef surfaces for lunch on the pier and then goes back to the reef. How far does he travel?



\Rightarrow

m

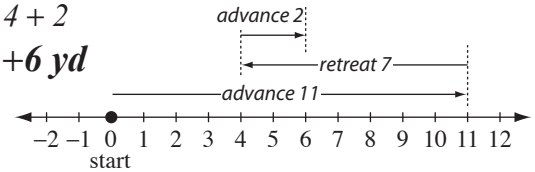
Skill 13.6 Solving word problems involving two or more integers.

- Start at the given point.
- Work in the given order.
- Visualize the position of the values on a number line.

Hint: Positive words: up above over forward advance gained earned later plus
Negative words: down below under backward retreat lost owed earlier minus

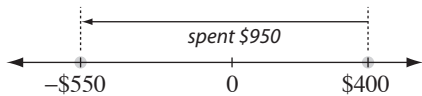
Q. During a football game the ball advanced 11 yd, retreated 7 yd and then advanced 2 yd. Where did the ball finish in relation to its starting point?

A. $Start: 0 + 11 - 7 + 2$
 $= 4 + 2$
 $= +6 \text{ yd}$



a) If Pip had \$400 and spent \$950, what is her bank balance?

$= 400 - 950 = \boxed{-\$550}$



b) Harry owed \$350. If he earned \$420, how much does Harry now have?

$= \dots = \boxed{}$

c) Chan owes \$420. If he earned \$280, what is Chan's bank balance?

$= \dots = \boxed{}$

d) Carbon dioxide boils at -78°C . At 21°C below this, carbon dioxide solidifies. At what temperature does carbon dioxide solidify?

$= \dots = \boxed{}^{\circ}\text{C}$

e) The Persians destroyed the original Acropolis in 480 B.C. Pericles rebuilt it 31 years later. What year was that?

$= \dots = \boxed{}$

f) Tutankhamun reigned for 9 years up until 1323 B.C. What year did Tutenkhamen come to the throne?

$= \dots = \boxed{}$

g) Oxygen boils at -183°C . At 35°C below this, oxygen solidifies. What is the temperature of solid oxygen?

$= \dots = \boxed{}^{\circ}\text{C}$

h) Helium boils at -269°C . At 3°C below this, helium solidifies. At what temperature does helium solidify?

$= \dots = \boxed{}^{\circ}\text{C}$

i) You bought \$1000 worth of stock. After the first year you lost \$480, but after the second year you gained \$220. What is the current value of your stock?

$= \dots = \boxed{}$

j) During a football game the ball advanced 2 yd, retreated 9 yd and then advanced 4 yd. Where did the ball finish in relation to its starting point?

$= \dots = \boxed{} \text{ yd}$

Skill 13.7 Adding integers.

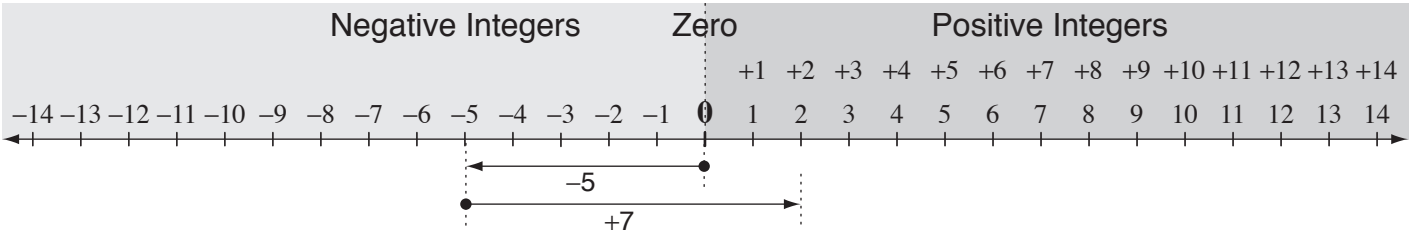
To add two integers with the **same sign**:

- Find their absolute values.
- Determine the sign of the result:
Use “+” if both integers are positive.
Use “-” if both integers are negative.
- **Add** the absolute values.

Hint: Every number has a sign attached to it, so if there is no sign, the number is positive. These signs should not be confused with the operations of addition and subtraction.

- The sign of the result of the addition can also be visualized using a number line.

Hint: ‘-’ means move left or backwards and ‘+’ means move right or forwards.



Q. $-5 + 7 =$

A. $-5 + 7 =$
 $= +(7 - 5)$
 $= 2$

$|-5| = 5, |7| = 7$
Integers have different signs and
 $|7| \geq |-5|$ so use “+”
Subtract $|7|$ and $|-5|$

a) $-2 + (-3) =$

both negative use “-”
 $|-2| = 2, |-3| = 3$
same signs, add
 $= -(2 + 3) = \boxed{-5}$

b) $-4 + 3 =$

$|-4| > |3|$, use “-”
different signs, subtract

.....
.....
..... =

c) $-8 + 6 =$

.....
.....
..... =

d) $8 + (-5) =$

.....
.....
..... =

e) $2 + (-6) =$

.....
.....
..... =

f) $5 + (-3) =$

.....
.....
..... =

g) $-2 + 4 =$

.....
.....
..... =

h) $9 + (-2) =$

.....
.....
..... =

i) $-4 + (-2) =$

.....
.....
..... =

j) $-8 + 3 =$

.....
.....
..... =

k) $-2 + (-6) =$

.....
.....
..... =

l) $-3 + (-6) =$

.....
.....
..... =

Skill 13.8 Subtracting integers.

- Consider subtracting an integer as adding its opposite.

To add two integers with the **same sign**:

- Find their absolute values.
- Determine the sign of the result:
Use “+” if both integers are positive.
Use “-” if both integers are negative.
- Add** the absolute values.

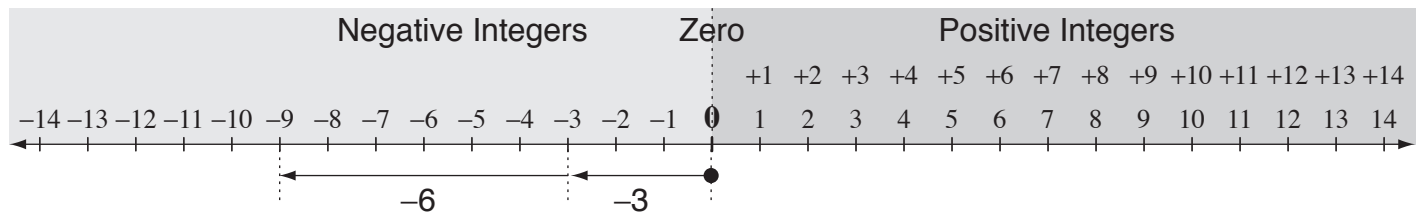
To add two integers with **different signs**:

- Find their absolute values.
- Determine the sign of the result:
Use the sign of the integer with the greater absolute value.
- Subtract** their absolute values.

Hint: Every number has a sign attached to it, so if there is no sign, the number is positive. These signs should not be confused with the operations of addition and subtraction.

- The sign of the result of the subtraction can also be visualized using a number line.

Hint: ‘-’ means move left or backwards and ‘+’ means move right or forwards.



Q. $-3 - 6 =$

start at -3, move backward 6

A. $-3 - 6 =$

$$= -3 + (-6)$$

$$= -(3 + 6)$$

$$= -9$$

Subtract 6 means add -6

$$|-3| = 3, |-6| = 6$$

Both integers negative so use “-”

Integers have same sign so add $|-3|$ and $|-6|$

a) $1 - 7 =$ *subtract 7 means add -7*

$$= +1 + (-7)$$

$$= -(7 - 1) = \boxed{-6}$$

$|-7| > |1|$, use “-”

b) $0 - 8 =$

$$=$$

$$= \boxed{}$$

c) $4 - 8 =$

$$=$$

$$= \boxed{}$$

d) $-3 - 5 =$

$$=$$

$$= \boxed{}$$

e) $-6 - 9 =$

$$=$$

$$= \boxed{}$$

f) $2 - (-1) =$

$$=$$

$$= \boxed{}$$

g) $3 - (-4) =$

$$=$$

$$= \boxed{}$$

h) $-8 - (-4) =$

$$=$$

$$= \boxed{}$$

i) $-2 - (-2) =$

$$=$$

$$= \boxed{}$$

Skill 13.9 Multiplying Integers.

MMBlue 1 1 2 2 3 3 4 4
MMGreen 1 1 2 2 3 3 4 4

- Multiply the absolute values.
- Determine the sign of the result:
If the numbers to be multiplied have the:

same sign: $++ = +$
 $-- = +$ positive result

different sign: $+- = -$
 $-+ = -$ negative result

Example: $-9 \times (-3) = 27$
 $-9 \times (-3)$
 $= 27$ (---+)

Example: $9 \times (-3) = -27$
 $9 \times (-3)$
 $= -27$ (+---)

Q. $-4 \times (-7) =$

A. $-4 \times (-7) = 28$
 $-4 \times (-7)$
 $= 28$ (---+)

$|-4| \times |-7| = 28$
 Same signs, both negative
 \Rightarrow positive result.

a) $-6 \times 7 =$ (-+--) b) $-2 \times 6 =$ c) $-8 \times 3 =$

d) $3 \times (-5) =$ e) $2 \times (-9) =$ f) $-8 \times (-8) =$

g) $-8 \times (-5) =$ h) $-9 \times 4 =$ i) $5 \times (-6) =$

j) $7 \times (-8) =$ k) $-4 \times 6 =$ l) $-7 \times 7 =$

m) $3 \times (-9) =$ n) $-7 \times (-4) =$ o) $-6 \times (-3) =$

p) $-4 \times (-4) =$ q) $5 \times (-9) =$ r) $-8 \times (-2) =$

s) $-5 \times (-5) =$ t) $-4 \times 5 =$ u) $-9 \times 9 =$

Skill 13.10 Dividing integers.

- Divide the absolute values.
- Determine the sign of the result:
If the numbers to be divided have the:

same sign: $++ = +$
 $-- = +$ positive result

different sign: $+- = -$
 $-+ = -$ negative result

Example: $-9 \div (-3) = 3$
 $-$ $-$ $+$ $+$ $-$ $-$ $+$

Example: $9 \div (-3) = -3$
 $+$ $-$ $-$ $-$ $+$ $-$ $-$

Q. $-30 \div 6 =$

A. $-30 \div 6 = -5$
 $-$ $+$ $-$ $-$

$|-30| \div |6| = 5$
Different signs,
 \Rightarrow negative result.

a) $12 \div (-4) = -3$ b) $27 \div (-3) =$ c) $-54 \div (-9) =$

d) $-72 \div (-12) =$ e) $-45 \div 9 =$ f) $-32 \div 8 =$

g) $-18 \div 2 =$ h) $-24 \div (-8) =$ i) $-63 \div 9 =$

j) $25 \div (-5) =$ k) $-56 \div (-7) =$ l) $-21 \div 7 =$

m) $-45 \div 5 =$ n) $-28 \div (-7) =$ o) $-54 \div 6 =$

p) $28 \div (-4) =$ q) $-35 \div (-7) =$ r) $-40 \div (-5) =$

s) $-36 \div 6 =$ t) $63 \div (-7) =$ u) $-36 \div 9 =$