

19. [Integers]

Skill 19.1 Recognizing integers.

MMBlue 11 22 33 44
MMGreen 11 22 33 44

The number 5 is read as **positive 5** and -5 as **negative 5**.

A positive number can be written with or without a “+” sign: $5 = +5$

INTEGERS are whole numbers, both positive and negative and including zero.

Zero is neither a negative nor a positive number.

INTEGERS can be used to describe things that have opposites:

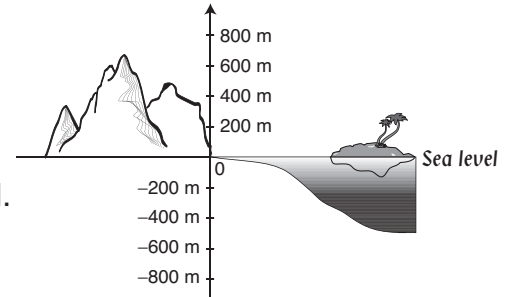
Altitude (above sea level compared to below sea level).

+ means above sea level.

An altitude is higher when it is further up from the sea level.

– means below sea level.

An altitude is lower when it is further down below the sea level.



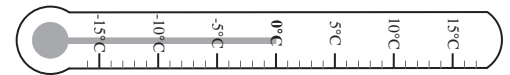
Temperature (above zero and below zero).

+ means above zero.

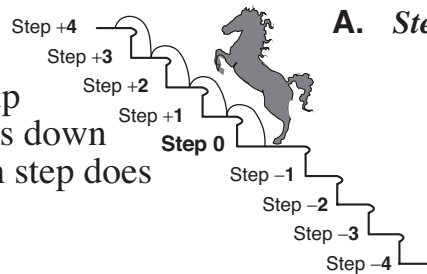
A temperature is higher when it is further to the right from 0°C .

– means below zero.

A temperature is lower when it is further to the left from 0°C .



Q. Starting on step 0, Rocky, the horse, jumps up 4 steps, then jumps down 6 steps. On which step does Rocky finish?



A. Step -2

When Rocky jumps up 4 steps from 0, he will then be on Step +4. From here he jumps down 6 steps. So, Rocky will finish on Step -2 .

Q. Which moment in time was the earliest:

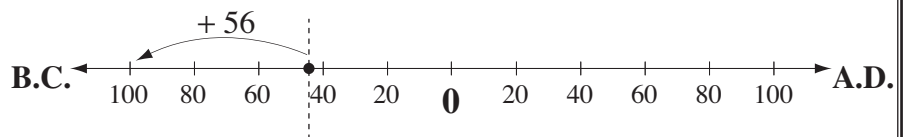
- A) 40 seconds before take-off,
- B) 50 seconds after take-off or
- C) 30 seconds before take-off?

A. A)

The time before take-off is always earlier than the time after take-off. So B) is ruled out. Compare 40 seconds before take-off to 30 seconds before take-off. The earliest is 40 seconds before take-off, or A).

Q. Julius Caesar was assassinated in 44 B.C., the year of his 56th birthday. In what year was he born?

A. 100 B.C. Use a time line.

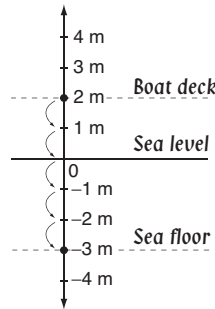


44 B.C.

Locate 44 B.C. on the line. Caesar was born 56 years before 44 B.C., so go back in time 56 years, starting from 44. This is the same as adding 44 and 56. $44 + 56 = 100$

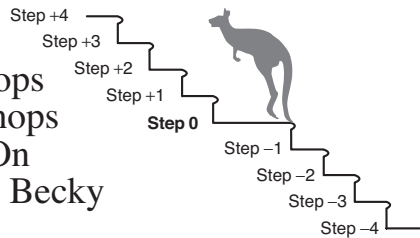
Q. A scuba diver jumps from a deck of a boat 2 m above the sea level and touches the sea floor 3 m below the sea level. How far below the deck is the sea floor?

A. 5 m Use a vertical number line.



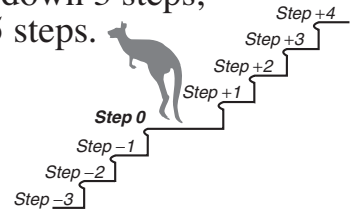
Locate 2 m above the sea level on the line.
Locate 3 m below the sea level (-3 m) on the line.
Count how many meters (units) are between the two points.
This is the same as adding 2 and 3
 $2 + 3 = 5$.

a) Starting on step 0, Becky, the kangaroo, hops up 1 step, then hops down 4 steps. On which step does Becky finish?



Step -3

b) Starting on step 0, Warwick, the wallaby, hops down 3 steps, then hops up 5 steps. On which step does Warwick finish?



c) Which temperature is the lowest:
A) 3°C above zero,
B) 10°C below zero or
C) 9°C below zero?

.....

d) Which altitude is the highest:
A) 180 m below sea level,
B) 1 m above sea level or
C) 27 m below sea level?

.....

e) Which temperature is the highest:
A) 15°C below zero,
B) 8°C below zero or
C) 3°C below zero?

.....

f) Which company recorded the worst quarterly results:
Company A: \$0.6 M loss,
Company B: \$1.2 M loss or
Company C: \$1.5 M profit?

.....

g) Cleopatra was born in 69 B.C. She followed Caesar to Rome in 46 B.C. How old was she then?

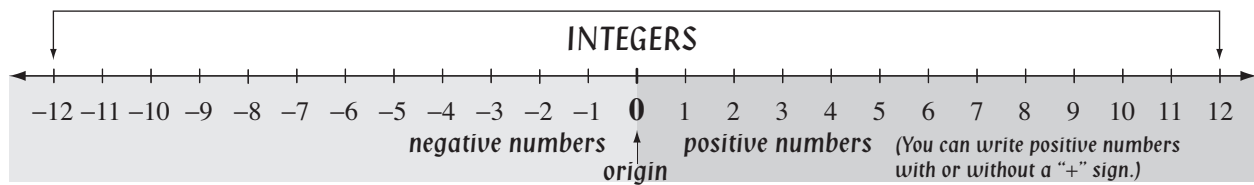
.....

h) The water in the Snake River rises from -2 m below the marker to +8 m. How much deeper is the river after the rise?

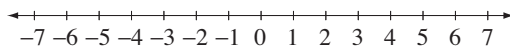
..... m

Skill 19.2 Representing integers on a number line.

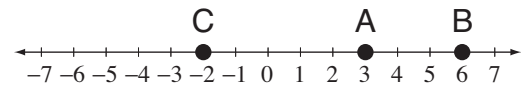
On a horizontal number line the positive numbers are to the right of the origin (zero) and the negative numbers are to the left of the origin.



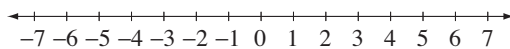
Q. On the number line below, mark the following points:
A at +3, B at +6 and C at -2.



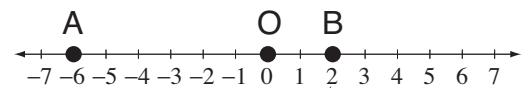
A.



Q. How many units from -6 to +2?



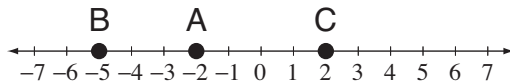
A. 8



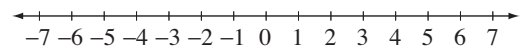
Label -6 with A, +2 with B and zero with O.
On the number line count how many units from A to B.

This is the same as adding 6 units from A to O and 2 units from O to B: $6 + 2 = 8$

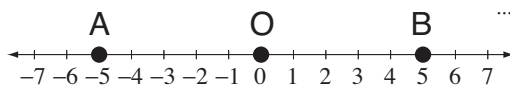
a) On the number line below, mark the following points:
A at -2, B at -5 and C at +2.



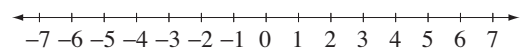
b) On the number line below, mark the following points:
A at -5, B at -1 and C at +3.



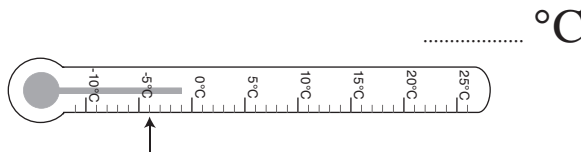
c) How many units from -5 to +5?



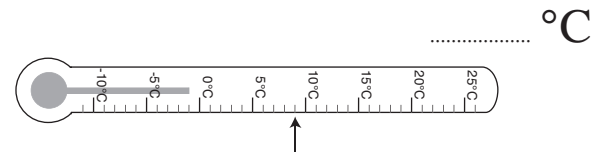
d) How many units from -6 to +4?



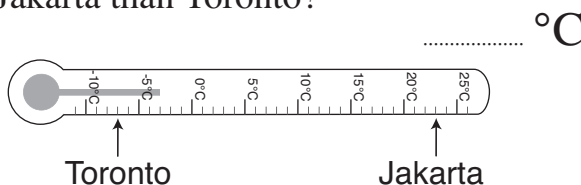
e) What is the temperature shown by the thermometer below?



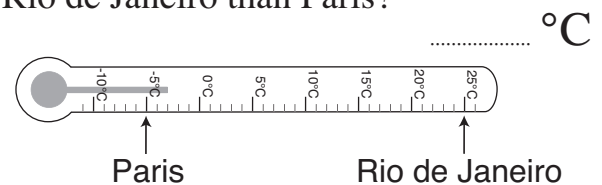
f) What is the temperature shown by the thermometer below?



g) How much warmer is it in Jakarta than Toronto?



h) How much warmer is it in Rio de Janeiro than Paris?



Skill 19.3 Comparing and ordering integers.

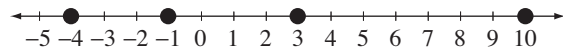
You can use a horizontal number line to compare and order integers. Numbers decrease as you move to the left on the horizontal number line and increase as you move to the right. A negative number is always smaller than a positive number.



Q. Arrange in order from smallest to largest:

10, 3, -4, -1

A. $-4, -1, 3, 10$

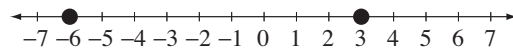


Mark the numbers on a horizontal number line. The numbers from smallest to largest are $-4, -1, 3, 10$. -4 is the smallest because it is the furthest to the left.

Q. Use $<$, $=$ or $>$ to complete the statement.

3 -6

A. $3 > -6$



Find -6 and 3 on the number line. As you move to the right on a number line, numbers have greater value so 3 is greater than -6 . We write $3 > -6$

a) Arrange in order from smallest to largest:

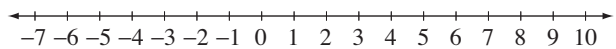
$-4, 3, -6, 2, 0$

$-6, -4, 0, 2, 3$



b) Arrange in order from largest to smallest:

$-1, 0, -3, 10, -7$



c) Arrange in order from coldest to warmest:

$-4^\circ\text{C}, -7^\circ\text{C}, 1^\circ\text{C}, -2^\circ\text{C}$

.....

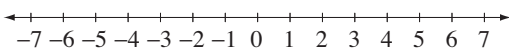
d) Arrange in order from earliest to latest:

18 B.C., 19 A.D., 13 B.C., 17 A.D.

.....

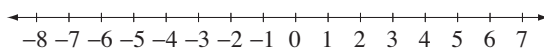
e) Use $<$, $=$ or $>$ to complete the statement.

-3 0



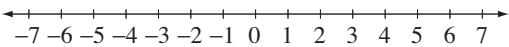
f) Use $<$, $=$ or $>$ to complete the statement.

-4 -8



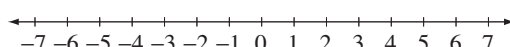
g) Use $<$, $=$ or $>$ to complete the statement.

7 -1



h) Use $<$, $=$ or $>$ to complete the statement.

-6 -1



Using a **number line** to add integers.

When adding integers, start at 0. If the first number is positive, move that many to the right on the number line but if the first number is negative move to the left.

From this point, continue moving right (+) or left (-) for the next numbers to be added.

OR

Using **rules** to add integers (positive and negative numbers).

Remember every number has a sign attached to it so if there's no sign the number is positive. These signs should not be confused with the operations of addition and subtraction. Every number with the sign that belongs to it is bracketed in the following examples so that you can see this idea.

If the numbers to be added have the

Rule 1: same sign - keep that sign and add the numbers.

$$\begin{aligned} (+4) + (+3) &= +(4 + 3) = +7 = 7 && \text{or simply } 4 + 3 = 7 \\ (-5) + (-8) &= -(5 + 8) = -13 && -5 + -8 = -13 \end{aligned}$$

Rule 2: different signs - use the sign of the larger number and then subtract the smaller number from the larger number.

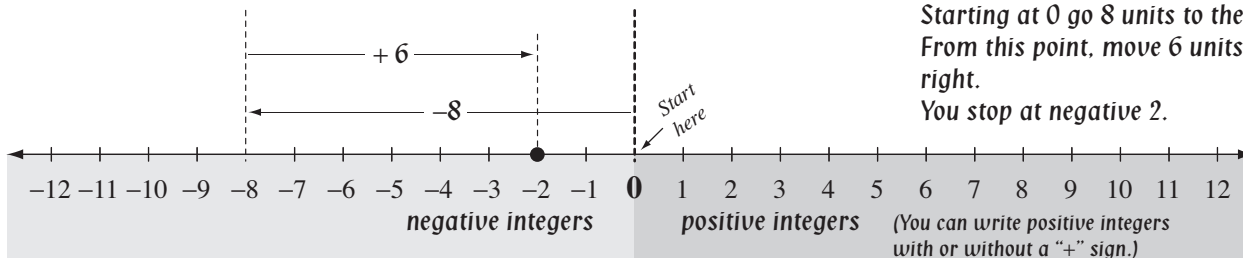
$$\begin{aligned} (-9) + (+3) &= -(9 - 3) = -6 && \text{or simply } -9 + 3 = -6 \\ (-4) + (+11) &= +(11 - 4) = +7 = 7 && -4 + 11 = 7 \end{aligned}$$

Q. $(-8) + (+6) =$

A. $(-8) + (+6) =$
 $= -(8 - 6)$
 $= -2$

Read as negative 8 plus positive 6 equals negative 2.
 (Note: Positive 6 can be read as 6.)

Starting at 0 go 8 units to the left.
 From this point, move 6 units to the right.
 You stop at negative 2.



a) $(-5) + (-4) =$
 $= -(5 + 4)$
 $= -9$

b) $(-9) + (-6) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

c) $(+5) + (-4) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

d) $(-2) + (+8) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

e) $(-10) + (+3) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

f) $(+7) + (-3) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

g) $(-1) + (-8) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

h) $(+6) + (-6) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

i) $(-4) + (+4) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

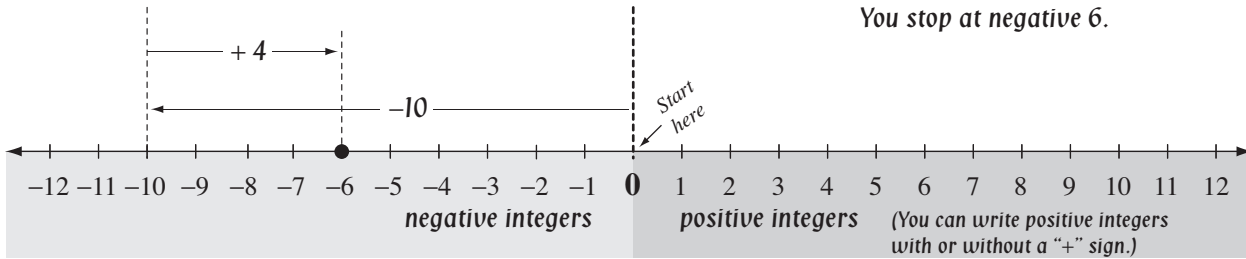
Skill 19.5 Subtracting integers.

Opposite numbers have the same value but different signs. They sum to zero. $(-5) + (+5) = 0$
To subtract an integer add its opposite, then follow the addition rules.

Q. $(-10) - (-4) =$

A. $(-10) - (-4) =$
 $= (-10) + (+4)$
 $= -(10 - 4)$
 $= -6$

Negative 10 take away negative 4
is the same as negative 10 plus positive 4.
OR
Using a number line:
Start at 0 and go 10 units to the left.
From this point, move 4 units to the right.
You stop at negative 6.



a) $(+6) - (-3) =$
 $= (+6) + (+3)$
 $= +(6 + 3)$
 $= +9$

b) $(-8) - (+5) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

c) $(-4) - (-9) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

d) $(-12) - (-3) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

e) $(-7) - (+4) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

f) $(+7) - (+3) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

g) $(-5) - (-2) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

h) $(+8) - (-3) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

i) $(-7) - (+9) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

j) $(+3) - (+10) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

k) $(+17) - (+8) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

l) $(+15) - (-6) =$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

<p>Q. The temperature at 6 A.M. was -5°C. By noon the temperature had risen 15°C. What was the temperature at noon?</p>	<p>A. $(-5) + (+15) =$ $= +(15 - 5)$ $= +10$ $= 10^{\circ}\text{C}$</p>	<p>Use integers to represent the two temperatures. At 6 A.M. -5°C At 12 P.M. 15°C more Add 15 to (-5). To do this, subtract the numbers and keep the + sign of the larger number.</p>
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<p>Q. Find the difference in altitude between the Dead Sea - Israel (1370 ft below sea level) and Mt Annapurna - Himalayas (26,540 ft above sea level).</p>	<p>A. $(+26,540) - (-1370) =$ $= 26,540 + 1370$ $= 27,910 \text{ ft}$</p>	<p>Use integers to represent the two altitudes. Dead Sea: -1370 ft Mt Annapurna: $+26,540 \text{ ft}$ Find the difference between $(+26,540)$ and (-1370).</p>
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a) At midnight the temperature was -10°C . Twelve hours later the temperature had risen 18°C . What was the midday temperature?

$$= (-10) + (+18)$$

$$= +(18 - 10)$$

$$= +8 \quad ^{\circ}\text{C}$$

b) At midnight the temperature was -24°C . Twelve hours later the temperature had risen 18°C . What was the midday temperature?

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots \quad ^{\circ}\text{C}$$

c) Climbing a mountain from 4500 ft to 22,500 ft, Gus observed the temperature drop from 20°C to -12°C . By how many degrees did the temperature fall?

$$= (+20) - (-12)$$

$$= (+20) + (+12)$$

$$= +(20 + 12)$$

$$= +32 \quad ^{\circ}\text{C}$$

d) How many years apart are January 1st, 35 B.C. and January 1st, 15 A.D.?

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots \text{ years}$$

e) The highest and lowest temperatures recorded in USA are 134°F (Death Valley, California, 1913) and -80°F (Prospect Creek, Alaska, 1971). What is the difference between these extremes?

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots \quad ^{\circ}\text{F}$$

f) Find the difference in altitude between the Mariana Trench - Pacific Ocean (36,200 ft below sea level) and Mt McKinley - Alaska (20,320 ft above sea level).

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots$$

$$= \dots\dots\dots \text{ ft}$$

Skill 19.7 Multiplying and dividing integers.Using **rules** to multiply and divide integers.

If the numbers to be multiplied or divided have the

Rule 1: same sign - the result is positive.

$$\begin{array}{l} (+) \times (+) \Rightarrow + \qquad \qquad (+) \div (+) \Rightarrow + \\ (-) \times (-) \Rightarrow + \qquad \text{and} \qquad (-) \div (-) \Rightarrow + \end{array}$$

Rule 2: different signs - the result is negative.

$$\begin{array}{l} (+) \times (-) \Rightarrow - \qquad \qquad (+) \div (-) \Rightarrow - \\ (-) \times (+) \Rightarrow - \qquad \qquad (-) \div (+) \Rightarrow - \end{array}$$

Q. $(-4) \times (+8) =$	A. $(-4) \times (+8)$ $= -32$	$4 \times 8 = 32$ and $(-) \times (+) \Rightarrow -$
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Q. $(-24) \div (-6) =$	A. $(-24) \div (-6)$ $= +4$ $= 4$	$24 \div 6 = 4$ and $(-) \div (-) \Rightarrow +$
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- | | | |
|---|--|--|
| a) $(-6) \times (-3) =$ <u> +18 </u> | b) $(-1) \times (+5) =$ | c) $(+2) \times (-3) =$ |
| d) $(-7) \times (-5) =$ | e) $(-5) \times (-4) =$ | f) $(-8) \times (+10) =$ |
| g) $(-20) \div (+2) =$ | h) $(+15) \div (-3) =$ | i) $(-16) \div (-4) =$ |
| j) $(-18) \div (+3) =$ | k) $(-30) \div (-6) =$ | l) $(+22) \div (-2) =$ |
| m) $(-12) \times (-4) =$ | n) $(-15) \times (+5) =$ | o) $(+10) \times (-10) =$ |
| p) $(-45) \div (-9) =$ | q) $(-64) \div (+8) =$ | r) $(+72) \div (-6) =$ |
| s) $(-6) \times (-6) =$ | t) $(-12) \times (+12) =$ | u) $(+75) \div (-5) =$ |