23. [Algebra - Equations]

Skill 23.1 Solving equations involving + or –

An equation is a number sentence that has an '=' symbol and an unknown number or quantity.
An example of an equation involving + or – is: $4 + \ldots\ldots = 12$ OR $4 + x = 12$

To solve an equation involving + or –:
Use **trial and error** to find the value of the unknown number (x) that will make the equation true.

$4 + x = 12$	Write the original equation.
$4 + \mathbf{8} = 12$	Trial substituting 8 for x.
$12 = 12$	The equation is true, so **8** is the unknown number.

OR

Use **inverse operations** and get x alone on one side of the equation to find its value. Addition and subtraction are inverse operations. Adding 4 and then subtracting 4 leaves a number unchanged. You can change one side of an equation in any way you like and the equation will stay true, as long as you make the same changes to the other side of the equation.

$4 + x = 12$	Write the original equation.
$4 + x - 4 = 12 - 4$	To get x alone, use the inverse operation to addition which is subtraction and subtract 4 from both sides.
$x = 8$	Simplify.

Q. $17 - \ldots\ldots = 5$

A. $17 - \ldots\ldots = 5$
$17 - \mathbf{12} = 5$

Think of different ways to say this.
17 take away the unknown number equals 5.
The missing number is 5 less than 17.
Use trial and error to make the equation true.

Q. $2 + \ldots\ldots = 6$

A. $2 + \ldots\ldots = 6$
$2 + \mathbf{4} = 6$
OR
$2 + \overset{?}{\ldots\ldots} = 6$
$2 + \overset{?}{\ldots\ldots} - 2 = 6 - 2$
$\overset{?}{\ldots\ldots} = \mathbf{4}$

Think of different ways to say this.
2 plus the unknown number equals 6.
The missing number is 2 less than 6.
Use trial and error to make the equation true.
OR
Get the missing number on its own. If 2 has been added to the missing number then do the inverse operation and subtract 2. Remember that you must do the same to the other side of the equation.

Q. $\ldots\ldots - 5 = 3$

A. $\ldots\ldots - 5 = 3$
$\mathbf{8} - 5 = 3$
OR
$\overset{?}{\ldots\ldots} - 5 = 3$
$\overset{?}{\ldots\ldots} - 5 + 5 = 3 + 5$
$\overset{?}{\ldots\ldots} = \mathbf{8}$

Think of different ways to say this.
5 subtracted from the unknown number is 3.
The missing number equals 3 plus 5.
Use trial and error to make the equation true.
OR
Get the missing number on its own. If 5 has been subtracted from the missing number then do the inverse operation and add 5. Remember that you must do the same to the other side of the equation.

a) $15 - \underline{8} = 7$

b) $22 - \underline{\quad} = 15$

c) $30 - \underline{\quad} = 14$

d) $3 + \underline{6} = 9$

$\underline{3 + ? - 3} = \underline{9 - 3}$

$\underline{?} = \underline{6}$

e) $\underline{\quad} - 5 = 4$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

f) $\underline{\quad} - 9 = 8$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

g) $4 + \underline{\quad} = 13$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

h) $\underline{\quad} + 7 = 11$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

i) $\underline{\quad} + 4 = 12$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

j) $5 + \underline{\quad} = 11$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

k) $8 + \underline{\quad} = 15$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

l) $\underline{\quad} - 7 = 6$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

m) $9 + \underline{\quad} = 24$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

n) $\underline{\quad} - 10 = 6$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

o) $6 + \underline{\quad} = 10$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

p) $8 + \underline{\quad} = 32$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

q) $\underline{\quad} - 7 = 3$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

r) $\underline{\quad} - 15 = 20$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

s) $12 + \underline{\quad} = 31$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

t) $\underline{\quad} - 24 = 24$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

u) $\underline{\quad} + 9 = 17$

$\underline{\quad} = \underline{\quad}$

$\underline{\quad} = \underline{\quad}$

Skill 23.2 Solving equations involving ×

An equation is a number sentence that has an '=' symbol and an unknown number or quantity.
An example of an equation involving + or – is: $4 \times \ldots\ldots = 12$ OR $4 \times x = 12$

To solve an equation involving ×:
Use **trial and error** to find the value of the unknown number (x) that will make the equation true.

$4 \times x = 12$	Write the original equation.
$4 \times \mathbf{3} = 12$	Trial substituting 3 for x.
$12 = 12$	The equation is true, so **3** is the unknown number.

OR

Use **inverse operations** and get x alone on one side of the equation to find its value.
Multiplication and division are inverse operations. Multiplying by 4 and then dividing by 4 leaves a number unchanged. You can change one side of an equation in any way you like and the equation will stay true, as long as you make the same changes to the other side of the equation.

$4 \times x = 12$	Write the original equation.
$4 \times x \div 4 = 12 \div 4$	To get x alone, use the inverse operation to multiplication which is division and divide both sides by 4.
$x = 3$	Simplify.

Q. $2 \times \ldots\ldots = 18$

A. $2 \times \ldots\ldots = 18$
$2 \times \mathbf{9} = 18$
OR
$2 \times \overset{?}{\ldots\ldots} = 18$
$2 \times \overset{?}{\ldots\ldots} \div 2 = 18 \div 2$
$\overset{?}{\ldots\ldots} = \mathbf{9}$

Think of different ways to say this.
2 multiplied by the unknown number equals 18.
The missing number is 18 less 2, how many times?
Use trial and error to make the equation true.
OR
Get the missing number on its own.
If 2 has been multiplied by the missing number then do the inverse operation and divide by 2.
Remember that you must do the same to both sides of the equation.

Q. $\ldots\ldots \times 5 = -30$

A. $\ldots\ldots \times 5 = -30$
$-\mathbf{6} \times 5 = -30$
OR
$\overset{?}{\ldots\ldots} \times 5 = -30$
$\overset{?}{\ldots\ldots} \times 5 \div 5 = (-30) \div 5$
$\overset{?}{\ldots\ldots} = -\mathbf{6}$

Think of different ways to say this.
The missing number, multiplied by 5 equals –30.
The missing number equals –30 divided by 5.
Use trial and error to make the equation true.
OR
Get the missing number on its own.
If the missing number has been multiplied by 5, then do the inverse operation and divide by 5.
Remember that you must do the same to the both sides of the equation.

a) $\ldots 3 \ldots \times 4 = 12$
$\ldots ?\times 4 \div 4 \ldots = \ldots 12 \div 4 \ldots$
$\ldots ? \ldots = \ldots 3 \ldots$

b) $2 \times \ldots\ldots = 16$
$\ldots\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

c) $\ldots\ldots \times 12 = 48$
$\ldots\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

d) ..... × 7 = 49
..... = ...
..... = ...

e) 8 × ..... = 40
..... = ...
..... = ...

f) ..... × 9 = 72
..... = ...
..... = ...

g) ..... × 4 = 36
..... = ...
..... = ...

h) ..... × 6 = 36
..... = ...
..... = ...

i) ..... × 7 = 63
..... = ...
..... = ...

j) 3 × ..... = 21
..... = ...
..... = ...

k) 15 × ..... = 30
..... = ...
..... = ...

l) ..... × 4 = 64
..... = ...
..... = ...

m) 3 × ..... = −18
..... = ...
..... = ...

n) ..... × 7 = −56
..... = ...
..... = ...

o) 11 × ..... = −77
..... = ...
..... = ...

p) (−8) × ..... = 96
..... = ...
..... = ...

q) (−12) × ..... = 84
..... = ...
..... = ...

r) ..... × (−16) = 48
..... = ...
..... = ...

s) (−25) × ..... = 125
..... = ...
..... = ...

t) ..... × (−10) = 200
..... = ...
..... = ...

u) ..... × 15 = −75
..... = ...
..... = ...

Skill 23.3 Solving equations involving fractions.

An equation is a number sentence that has an '=' symbol and an unknown number or quantity.
An example of an equation involving fractions is: $\frac{1}{4}$ of ..... $= 3$ OR $\frac{1}{4}$ of $x = 3$

To solve an equation involving fractions:
Use **trial and error** to find the value of the unknown number (x) that will make the equation true.

$\frac{1}{4}$ of $x = 3$	Write the original equation and remember "of" means "×"
$\frac{1}{4} \times 12 = 3$	Substitute 12 for x and multiply $\frac{1}{4}$ by 12.
$3 = 3$	The equation is true, so **12** is the solution.

OR
Use **inverse operations** and get x alone on one side of the equation to find its value.
Multiplication and division are inverse operations. Dividing by 4 and then multiplying by 4 leaves a number unchanged. You can change one side of an equation in any way you like and the equation will stay true as long as you make the same changes to the other side of the equation.

$\frac{1}{4}$ of $x = 3$	Write the original equation.
$\frac{1}{4} \times x \times 4 = 3 \times 4$	To get x alone, use the inverse operation to division which is multiplication and multiplying by 4 on both sides.
$x = 12$	Simplify.

Q. $\frac{1}{2}$ of ..... $= 6$

A.
$\frac{1}{2}$ *of* ..... $= 6$
$\frac{1}{2}$ *of* ***12*** $= 6$
OR
$\frac{1}{2} \times$ *?* $= 6$
$\frac{1}{2} \times$ *?* $\times 2 = 6 \times 2$
? $=$ ***12***

Think of different ways to say this.
Half of the missing number equals 6.
The missing number is 2 lots of 6.
Use trial and error to make the equation true.
OR
Get the missing number on its own. Multiplying by $\frac{1}{2}$ is the same as dividing by 2. If the missing number has been divided by 2, then do the inverse operation and multiply by 2. Remember that you must do the same to both sides of the equation.

Q. $\frac{2}{3}$ of ..... $= -10$

A.
$\frac{2}{3}$ *of* ..... $= -10$
$\frac{2}{3}$ *of* $(-15) = -10$
OR
$\frac{2}{3} \times$ *?* $= -10$
$\frac{2}{3} \times$ *?* $\times 3 = (-10) \times 3$
$2 \times$ *?* $= (-30)$
$2 \times$ *?* $\div 2 = (-30) \div 2$
? $=$ ***−15***

Think of different ways to say this.
Two thirds of the missing number is −10.
One third of the missing number is half of −10.
Use trial and error to make the equation true.
OR
Get the missing number on its own. Multiplying by $\frac{2}{3}$ is the same as dividing by 3 and then multiplying by 2. If the missing number has been divided by 3 and then multiplied by 2, then do the inverse operations and multiply by 3 and then divide by 2. Remember that you must do the same to both sides of the equation.

a) $\frac{1}{3}$ of 18 ..... $= 6$
$\frac{1}{3} \times ? \times 3 = 6 \times 3$
? $= 18$

b) $\frac{1}{2}$ of ..... $= 7$
..... $=$ .....
..... $=$ .....

c) $\frac{1}{5}$ of ..... $= 5$
..... $=$ .....
..... $=$ .....

d) $\frac{1}{4}$ of ..... = 11

..... = ...

..... = ...

e) $\frac{1}{7}$ of ..... = 4

..... = ...

..... = ...

f) $\frac{1}{6}$ of ..... = 8

..... = ...

..... = ...

g) $\frac{1}{3}$ of ..... = 8

..... = ...

..... = ...

h) $\frac{1}{5}$ of ..... = 9

..... = ...

..... = ...

i) $\frac{1}{7}$ of ..... = 6

..... = ...

..... = ...

j) $\frac{3}{5}$ of 10 = 6

$\frac{3}{5} \times ? \times 5$ = 6×5

$3 \times ? \div 3$ = $30 \div 3$

? = 10

k) $\frac{3}{4}$ of ..... = 12

..... = ...

..... = ...

..... = ...

l) $\frac{2}{3}$ of ..... = 14

..... = ...

..... = ...

..... = ...

m) $\frac{3}{7}$ of ..... = –6

..... = ...

..... = ...

..... = ...

n) $\frac{4}{5}$ of ..... = –8

..... = ...

..... = ...

..... = ...

o) $\frac{3}{8}$ of ..... = –6

..... = ...

..... = ...

..... = ...

p) $\frac{2}{5}$ of ..... = –4

..... = ...

..... = ...

..... = ...

q) $\frac{5}{6}$ of ..... = –10

..... = ...

..... = ...

..... = ...

r) $\frac{2}{9}$ of ..... = –6

..... = ...

..... = ...

..... = ...

Skill 23.4 Solving equations involving +, –, × and ( ).

Q. $2 \times (8 - \ldots\ldots) = 6$

A. $2 \times (8 - \ldots\ldots) = 6$
$2 \times 3 = 6$
$8 - 5 = 3$
$2 \times (8 - \mathbf{5}) = 6$

For simplicity consider the equation within the brackets as one number.
Think of different ways to say this.
2 multiplied by (a number) equals 6.
The missing number equals 6 divided by 2.
The number is 3. The equation within the brackets equals 3.
Think of different ways to say the newly formed mathematical sentence.
8 take away the missing number equals 3.
The missing number equals 8 take away 3.

Q. $3 + 2 \times \ldots\ldots = 7$

A. $3 + 2 \times \ldots\ldots = 7$
$3 + 4 = 7$
$3 + 2 \times \mathbf{2} = 7$

Use trial and error. Remember the order of operations: ( ), then ×, before + and –

2 multiplied by 1 equals 2. Adding 3 to 2 you get 5. Not correct.
2 multiplied by 2 equals 4. Adding 3 to 4 you get 7. Correct.

a) $2 \times (9 - \underline{5}) = 8$
$2 \times 4 = 8$
$9 - 5 = 4$

b) $3 \times (15 - \ldots\ldots) = 9$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

c) $4 \times (9 - \ldots\ldots) = 28$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

d) $3 \times (8 - \ldots\ldots) = 6$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

e) $4 \times (11 - \ldots\ldots) = 16$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

f) $5 \times (8 - \ldots\ldots) = 20$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

g) $2 + 2 \times \underline{6} = 14$
$2 + 12 = 14$
$2 \times 6 = 12$

h) $3 + 4 \times \ldots\ldots = 23$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

i) $7 + 3 \times \ldots\ldots = 16$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

j) $4 + 3 \times \ldots\ldots = 19$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

k) $5 + 4 \times \ldots\ldots = 13$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

l) $8 + 5 \times \ldots\ldots = 28$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

m) $\underline{6} \times 2 + 6 = 18$
$12 + 6 = 18$
$\ldots\ldots = \ldots\ldots$

n) $2 \times \ldots\ldots - 7 = 9$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

o) $3 \times \ldots\ldots - 6 = 15$
$\ldots\ldots = \ldots\ldots$
$\ldots\ldots = \ldots\ldots$

Skill 23.5 Solving equations involving decimals.

Q. $4 - \ldots\ldots = 1.2$

A. $4 - \ldots\ldots = 1.2$
$4 - \mathbf{2.8} = 1.2$

Think of different ways to say this.
4 take the unknown number equals 1.2
The missing number is 1.2 less than 4.
Use trial and error to make the equation true.

Q. $5.4 + \ldots\ldots = 6$

A. $5.4 + \ldots\ldots = 6$
$5.4 + \mathbf{0.6} = 6$
OR
$5.4 + \overset{?}{\ldots\ldots} = 6$
$5.4 + \overset{?}{\ldots\ldots} - 5.4 = 6 - 5.4$
$\overset{?}{\ldots\ldots} = \mathbf{0.6}$

Think of different ways to say this.
5.4 plus the missing number equals 6.
The missing number is 5.4 less than 6.
Use trial and error to make the equation true.
OR
Get the missing number on its own.
If 5.4 has been added to the missing number then do the inverse operation and subtract 5.4. Remember that you must do the same to the other side of the equation.

a) $5.4 - \ldots\ldots = 3$

b) $8.5 - \ldots\ldots = 8$

c) $10 - \ldots\ldots = 5.5$

d) $3.8 + \underline{0.2} = 4$
$3.8 + ? - 3.8 = 4 - 3.8$
$? = 0.2$

e) $4.1 + \ldots\ldots = 10$

f) $1.1 \times \ldots\ldots = 3.3$

g) $6.5 + \ldots\ldots = 9$

h) $0.5 \times \ldots\ldots = 10$

i) $5.5 + \ldots\ldots = 10$

j) $7.3 + \ldots\ldots = 9$

k) $2.9 + \ldots\ldots = 6$

l) $1.2 \times \ldots\ldots = 12$

m) $2.5 \times \ldots\ldots = 5$

n) $1.4 \times \ldots\ldots = 4.2$

o) $1.5 \times \ldots\ldots = 6$

Skill 23.6 Solving equations to find the value of *x*.

Q. If $2x = 14$, find x

A.
$2x = 14$
$2 \times x = 14$
$2 \times x \div 2 = 14 \div 2$
$x = 7$

2 multiplied by what number equals 14?
Try algebra as shown or use trial and error.
Start with what you know.
2 multiplied by 5 equals 10. 10 is less than 14 so try 2 multiplied by 6.
In fact 2 multiplied by 7 equals 14 so x is 7.

Q. If $2x + 1 = 13$, find x

A.
$2x + 1 = 13$
$2 \times x - 1 = 13 - 1$
$2 \times x = 12$
$x = 12 \div 2$
$x = 6$

What number, multiplied by 2 and then having 1 added, equals 13?
Try algebra as shown or use trial and error.
Start with what you know.
2 multiplied by 5 equals 10. 10 is 3 less than 13 so try 2 multiplied by 6.
In fact 2 multiplied by 6 plus 1 equals 13 so x is 6.

a) If $4x = 24$, find x

$4 \times x = 24$
$4 \times x \div 4 = 24 \div 4$
$x = 6$

b) If $8x = 16$, find x

c) If $5x = 20$, find x

d) If $3x = -9$, find x

$3 \times x = -9$
$3 \times x \div 3 = (-9) \div 3$
$x = -3$

e) If $6x = 30$, find x

f) If $7x = 21$, find x

g) If $4x + 5 = 21$, find x

$4 \times x + 5 = 21$
$4 \times x + 5 - 5 = 21 - 5$
$4 \times x \div 4 = 16 \div 4$
$x = 4$

h) If $2x + 3 = 19$, find x

i) If $5x - 9 = 11$, find x

j) If $3x + 12 = 0$, find x

$3 \times x + 12 = 0$
$3 \times x + 12 - 12 = 0 - 12$
$3 \times x \div 3 = (-12) \div 3$
$x = -4$

k) If $5x - 4 = -34$, find x

l) If $6x + 5 = -37$, find x