

12. [Exploring Number]

Skill 12.1 Using "order of operations" involving a mix of (), ×, ÷, + or -

MMMaive 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Follow this order of operations:
Simplify inside the brackets.
Multiply (×) and/or divide (÷) in order from left to right.
Add (+) and/or subtract (-) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$
 $= 18 \div 6 + 2$ *simplify the brackets first*
 $= 3 + 2$ *division before addition*
 $= 5$

a) $6 + 12 \div 4 \times 3 =$

$= 6 + 3 \times 3$
 $= 6 + 9 = \boxed{}$

b) $6 \times 15 - 8 \times 3 =$

$= $
 $= = \boxed{}$

c) $5 + 12 \div 6 \times 3 =$

$= $
 $= = \boxed{}$

d) $3 \times (5 - 3) \times 8 =$

$= $
 $= = \boxed{}$

e) $(15 + 8) - (7 + 6) =$

$= $
 $= = \boxed{}$

f) $120 \div 5 - 6 \times 3 =$

$= $
 $= = \boxed{}$

g) $7 \times 8 + 24 \div 2 - 5 \times 6 =$

$= $
 $= = \boxed{}$

h) $120 \div (2 + 5 \times 2) - 9 =$

$= $
 $= = \boxed{}$

i) $100 - 5 \times (4 + 8) \div 4 - 55 =$

$= $
 $= = \boxed{}$

j) $13 + 6 \times 3 - 5 \times (2 + 4) =$

$= $
 $= = \boxed{}$

k) $27 - 3 \times (5 + 7) \div 6 + 10 =$

$= $
 $= = \boxed{}$

l) $5 + 3 \times 9 - 3 \times (2 + 8) =$

$= $
 $= = \boxed{}$

- Follow this order of operations:
Simplify inside the brackets.
Evaluate all powers.
Multiply (×) and/or divide (÷) in order from left to right.
Add (+) and/or subtract (–) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$
 $= (6 + 10)^2$ *multiply within brackets first*
 $= 16^2$ *add with the brackets*
 $= 256$

a) $(3 + 5)^2 =$
 $= 8^2$ =

b) $(12 - 7)^2 =$
 $=$ =

c) $(5 + 5 \times 3)^2 =$
 $=$
 $=$ =

d) $(2 \times 4 + 6)^2 =$
 $=$
 $=$ =

e) $(2 + 8)^2 \div 4 =$
 $=$
 $=$ =

f) $(7 + 5)^2 \div 8 =$
 $=$
 $=$ =

g) $5 + (12 - 6)^2 =$
 $=$
 $=$ =

h) $8 + (13 - 8)^2 =$
 $=$
 $=$ =

i) $(4 \times 2 + 2)^2 =$
 $=$
 $=$ =

j) $(3 \times 4 + 8)^2 =$
 $=$
 $=$ =

k) $(10 - 6)^2 \div (20 - 18) =$
 $=$
 $=$ =

l) $(10 - 5)^2 \div (14 - 9) =$
 $=$
 $=$ =

To round terminating decimals to a given place:

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
 5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

To round repeating decimals to a given place:

- Write the first 4 digits after the decimal point. (see skill 7.7, page 77)
- Apply the procedure described above for terminating decimals.

Q. Round $0.4\bar{6}$ to the nearest thousandth.

A. $0.4\bar{6} = 0.4666\dots$

6 is repeating indefinitely

0.4666... circle the fourth digit

≈ 0.467 6 ≥ 5 round up by adding 1 to 6

a) Round 0.13 to the nearest tenth.

0.13 $3 < 5$ round down by keeping 1 0.1

b) Round 7.89 to the nearest tenth.

.....

c) Round 12.45 to the nearest tenth.

.....

d) Round 31.5841 to the nearest hundredth.

31.5841 $4 < 5$ round down by keeping 8 31.58

e) Round 24.793 to the nearest hundredth.

.....

f) Round 4.231 to the nearest hundredth.

.....

g) Round 3.859 to the nearest tenth.

.....

h) Round 50.296 to the nearest hundredth.

.....

i) Round $4.\bar{7}$ to the nearest hundredth.

$4.\bar{7} = 4.7777\dots$ $7 \geq 5$ round up by adding 1 to 7 \approx 4.78

j) Round $3.\bar{42}$ to the nearest hundredth.

..... \approx

k) Round $0.\bar{6}$ to the nearest hundredth.

..... \approx

l) Round $1.\bar{73}$ to the nearest thousandth.

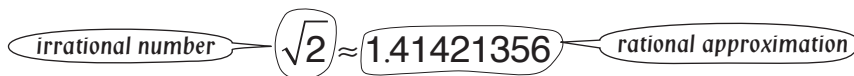
..... \approx

m) Round $4.2\bar{8}$ to the nearest thousandth.

..... \approx

n) Round $0.\bar{16}$ to the nearest thousandth.

..... \approx



- Circle the digit to the right of the requested place.
 - If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.
- Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the nearest hundredth.*

Q. $\cos 45^\circ \approx 0.70711$
Write the rational approximation of $\cos 45^\circ$ correct to two decimal places.

A. 0.70711
circle the third digit

0.71
 $7 \geq 5$
round up by adding 1 to 0

a) $\sqrt{12} \approx 3.46410162$
Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

3.46410162
 $4 < 5$
round down by keeping 6
 ≈ 3.46

b) $\sqrt{20} \approx 4.47213595$
Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

..... \approx

c) $\sqrt{24} \approx 4.89897949$
Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

..... \approx

d) $\sqrt{30} \approx 5.47722558$
Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

..... \approx

e) $\pi \approx 3.14159265$
Write the rational approximation of π correct to three decimal places.

..... \approx

f) $\phi \approx 1.61803398$
Write the rational approximation of ϕ correct to three decimal places.

..... \approx

g) $\sin 15^\circ \approx 0.25882$
Write the rational approximation of $\sin 15^\circ$ correct to three decimal places.

..... \approx

h) $\tan 60^\circ \approx 1.73205$
Write the rational approximation of $\tan 60^\circ$ correct to three decimal places.

..... \approx

i) $e \approx 2.71828182$ (Euler's number)
Write the rational approximation of e correct to two decimal places.

..... \approx

j) $\sqrt{10} \approx 3.16227766$
Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

..... \approx

| | | |
|--|---|---|
| 2.43×10^5 Scientific Notation Product of: Number ≥ 1 and < 10 Power of 10 with positive exponent | = | $243,000$ Standard Form Very large |
|--|---|---|

| | | |
|---|---|--|
| 8.02×10^{-4} Scientific Notation Product of: Number ≥ 1 and < 10 Power of 10 with negative exponent | = | 0.000802 Standard Form Very small |
|---|---|--|

If the power of 10 is **positive**:

- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary.
 Example: $3.1 = 3.1000$
 Hint: By convention $37 = 37. = 37.0$

If the power of 10 is **negative**:

- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary.
 Example: $4.5 = 00004.5$
- If the result is less than 1, write a zero in the units place.
 Hint: By convention 0.37 not $.37$

| | |
|--|--|
| <p>Q. Write 3.5×10^{-4} m, the diameter of optical fiber, in standard form.</p> | <p>A. 3.5×10^{-4} <small>exponent = -4</small> $= 00003.5 \times 10^{-4}$ <small>move decimal point 4 places left</small> $= 0.00035$ <small>add zeros as place holders</small></p> |
|--|--|

- a)** 6.2×10^5 is the scientific notation for:
 A) 6200 B) 620,000 C) 6.20000

$6.2 \times 10^5 =$ exponent = +5
 $= 620,000.00$ 5 places right B

- b)** 4.12×10^6 is the scientific notation for:
 A) 4,120,000 B) 412,000 C) 4.120000

$=$

- c)** Earth's atmosphere extends upward for 9.65×10^5 m. Write this in standard form.

$.....$

- d)** Write 1.3×10^9 , China's population in 2006, in standard form.

$.....$

- e)** The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
 A) 0.0008 B) 8000 C) 0.008

$=$

- f)** The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
 A) 0.00025 B) 0.000025 C) 250,000

$=$

- g)** Write 2.5×10^{-11} m, the radius of a hydrogen atom, in standard form.

$.....$

- h)** Write 5×10^{-7} m, the size of a speck of dust, in standard form.

$.....$

- Follow this order of operations:
Simplify inside the brackets.
Multiply (\times) and/or divide (\div) in order from left to right.
Add ($+$) and/or subtract ($-$) in order from left to right.

Q. $-9 \div 3 - 5 \times (-10) =$

A. $-9 \div 3 - 5 \times (-10) =$

$= -3 - (-50)$

divide and multiply first

$= -3 + 50$

$= 47$

subtract, use "+"

a) $(12 - 2) \div (9 - 11) =$

$= 10 \div (-2) = \boxed{-5}$

b) $-56 \div 8 - 4 \times 7 =$

$= \dots = \boxed{}$

c) $-12 \div 4 + (-3) \times 4 =$

$= \dots = \boxed{}$

d) $(-15 + 11) \times (9 - 14) =$

$= \dots = \boxed{}$

e) $-8 \times 6 - 42 \div 7 =$

$= \dots = \boxed{}$

f) $(9 - 17) \div (-6 + 8) =$

$= \dots = \boxed{}$

g) $(13 + 11) \times (7 - 10) =$

$= \dots = \boxed{}$

h) $-3 \times 7 - 12 \div 6 =$

$= \dots = \boxed{}$

i) $15 - 8 \times (10 - 5) \div 4 - 12 =$

$= \dots = \boxed{}$

j) $-7 + 12 \div (4 + 8) \times 6 + 10 =$

$= \dots = \boxed{}$

k) $-4 + 2 \times (8 - 12) \div 4 - 12 =$

$= \dots = \boxed{}$

l) $1 - 16 \div (3 + 5) \times 3 - 15 =$

$= \dots = \boxed{}$

m) $-10 \times (14 - 7) =$

$= \dots = \boxed{}$

n) $-28 \div (-1 - 6) + 17 =$

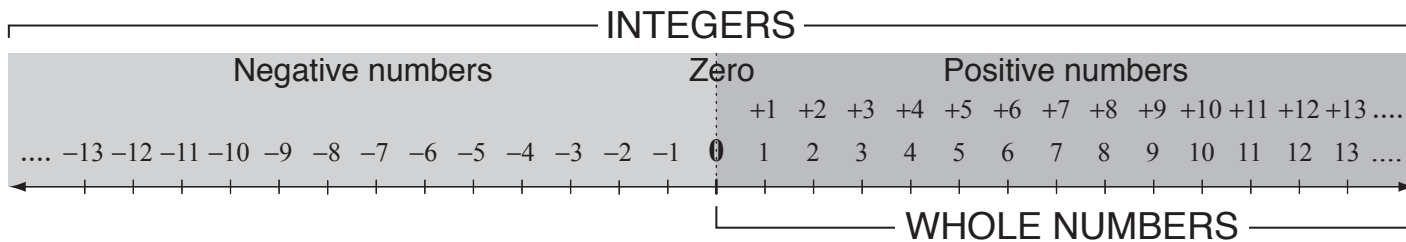
$= \dots = \boxed{}$

o) $-9 - 4 \times (13 - 2) =$

$= \dots = \boxed{}$

p) $-9 - 2 \times (19 - 3) =$

$= \dots = \boxed{}$



- Decide if a number is a whole number or an integer, based on their definition and hints below. (see Glossary)

Hints: Negative numbers, fractions, terminating decimals, repeating decimals and infinite non-repeating decimals are not whole numbers.

Any fraction whose numerator is divisible by the denominator is a whole number: $\frac{12}{4} = 3$

Any decimal with only zeros after the decimal point is a whole number: $8.00 = 8$

Fractions, terminating decimals, repeating decimals and infinite non-repeating decimals are not integers.

Any fraction whose numerator is divisible by the denominator is an integer: $\frac{5}{1} = 5$, $\frac{12}{4} = 3$

Any decimal with only zeros after the decimal point is an integer: $8.00 = 8$

Any square root of a perfect square is an integer: $\sqrt{16} = 4$

- Q.** Choose the whole numbers from this list:

$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

- A.** -7 is negative, so not a whole number

$\frac{8}{2} = 8 \div 2 = 4$ is a whole number

$-\frac{1}{3}$ is a fraction, so not a whole number

-3.6 is a decimal, so not a whole number

So $\frac{8}{2}$, **0**, **50** are whole numbers.

- a)** Choose the whole numbers from this list:

$7.43, \textcircled{89}, -5, 3\frac{1}{5}, \textcircled{14}, 0.6$

- b)** Choose the whole numbers from this list:

$567, 0.73, -4, \frac{3}{10}, 12, 0$

- c)** Choose the whole numbers from this list:

$1.4142, 18, -5.\bar{9}, \frac{4}{11}, -5, 143$

- d)** Choose the whole numbers from this list:

$-25, 0.6666\dots, 34, \frac{5}{7}, -1, 8.93567$

- e)** Choose the integers from this list:

$-3.5, 11, 2.\bar{14}, -1, 3\frac{2}{7}, 2$

- f)** Choose the integers from this list:

$3.14, \frac{16}{4}, -3, -0.\bar{72}, \sqrt{25}$

- g)** Choose the integers from this list:

$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

- h)** Choose the integers from this list:

$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

A number is **rational** if:

- It can be written as a fraction (ratio) of two integers.

Hint: All integers are rational numbers: $-2, 700, \sqrt{16}, \frac{5}{1}, \frac{25}{5}$

All terminating decimals are rational numbers: $2.16, -5.753469$

All repeating decimals are rational numbers: $0.57575757\dots = 0.\overline{57}$

Q. Which numbers are rational?

- A) $-\sqrt{\frac{3}{5}}$
- B) $0.999\dots$
- C) $0.12357102\dots$
- D) $\frac{11}{2}$

A. $-\sqrt{\frac{3}{5}}$ is not rational, because $\frac{3}{5}$ is not a perfect square.

$0.999\dots$ is rational, because it is a repeating decimal.

$0.12357102\dots$ is not rational, because it has infinite non-repeating digits after the decimal point.

$\frac{11}{2}$ is rational, because it is a fraction.

So **B and D** are rational.

a) Choose the rational numbers from this list:

$1.41421356\dots, \sqrt{\frac{1}{4}}, -\frac{1}{3}, 2.18, \frac{\pi}{6}$

b) Choose the rational numbers from this list:

$-5.\overline{2}, \frac{3\pi}{2}, \sqrt{10}, 3.14, \frac{659}{3867}$

c) Which numbers are rational?

- A) $\sqrt{\frac{4}{9}}$
- B) $-\frac{\pi}{4}$
- C) $3.14159265\dots$
- D) 1.75

A and D

d) Which numbers are **not** rational?

- A) $3.4\overline{8}$
- B) $1.61803399\dots$
- C) $\frac{\pi}{2}$
- D) $\sqrt{16}$

e) Which numbers are rational?

- A) $\frac{21}{55}$
- B) 0.00007
- C) $6.9205729744\dots$
- D) $-\sqrt{6}$

f) Which numbers are rational?

- A) $\sqrt{10}$
- B) $77.\overline{7}$
- C) $-\frac{11}{2}$
- D) $\sqrt{\frac{3}{7}}$

g) Which numbers are rational?

- A) $\sqrt{8}$
- B) $6.5\overline{9}$
- C) $-4.131133111333\dots$
- D) $3.161616\dots$

h) Which numbers are **not** rational?

- A) $-0.315315315\dots$
- B) $\sqrt{3}$
- C) $2.135791113\dots$
- D) $\frac{11}{49}$

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-repeating digits after the decimal point: 2.52849302953...

Hint: Square roots of prime numbers or rational numbers that are not perfect squares are

irrational numbers: $\sqrt{5}$, $\sqrt{18}$

Special numbers, such as π , e , ϕ are irrational.

Some values of trigonometric and logarithmic functions are irrational.

Q. Which is an irrational number?

- A) $\sqrt{900}$ B) $\frac{1}{20}$
C) $-\sqrt{38}$ D) -5.75

A. $\sqrt{900}$ is rational, because it equals 30.

$\frac{1}{20}$ is rational, because it is a fraction.

$\sqrt{38}$ is irrational, because it is a square root of a rational number that is not a perfect square.

5.75 is rational, because it is a decimal.

So **C** is an irrational number.

a) Which is an irrational number?

- A) ϕ B) 120
C) $\sqrt{25}$ D) -0.1675

A

b) Which is an irrational number?

- A) 0 B) $-5.636363\dots$
C) $\frac{3}{17}$ D) $\frac{1}{\sqrt{2}}$

c) Which is an irrational number?

- A) 3 B) -2.5
C) $\sqrt{2}$ D) $-\sqrt{4}$

d) Which is an irrational number?

- A) $2.\bar{6}$ B) 6.15
C) $\sqrt{7}$ D) $5\frac{3}{10}$

e) Which is an irrational number?

- A) $\frac{659}{3867}$ B) 2.7182813...
C) $-9.\bar{42}$ D) $\sqrt{\frac{9}{4}}$

f) Which is an irrational number?

- A) 3.15315315... B) 5001
C) $\frac{3}{10}$ D) $\sqrt{18}$

g) Choose the irrational numbers from this list:

$\sqrt{15}$, $-\frac{2}{3}$, -6 , $0.13133133313\dots$, $\sqrt{5}$

h) Choose the irrational numbers from this list:

$-\frac{3}{14}$, $\sqrt{\frac{3}{5}}$, π , $\frac{53}{83}$, $0.12357102\dots$

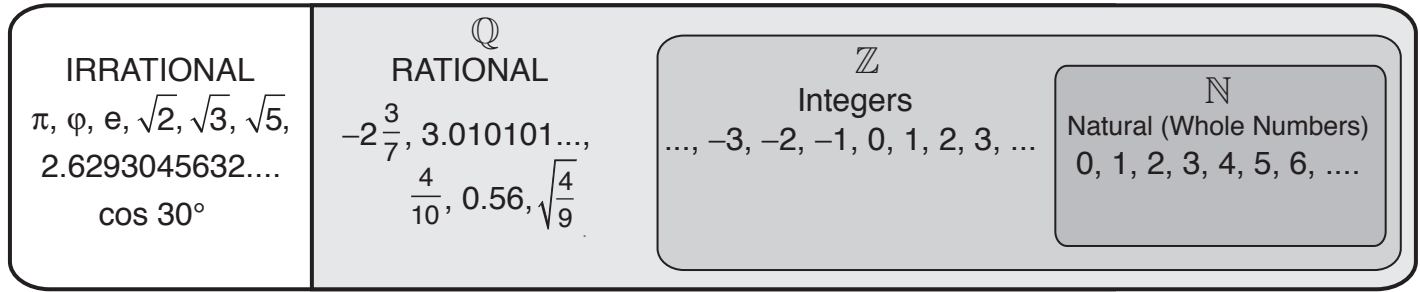
i) Choose the irrational numbers from this list:

$\sqrt{30}$, $0.\bar{67}$, $6.921921921\dots$, $-\sqrt{3}$, $\frac{\pi}{2}$

j) Choose the irrational numbers from this list:

$1\frac{1}{17}$, -45 , $3.14159\dots$, $\sqrt{\frac{6}{7}}$, ϕ

\mathbb{R} REAL NUMBERS



Hint: Rational numbers include integers, terminating decimals and repeating decimals.
 Irrational numbers include infinite non-repeating decimals.

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
 Irrational numbers $\subset \mathbb{R}$

Q. Which classes of numbers describe $-\sqrt{81}$? **A.** $-\sqrt{81} = -9$

| | | |
|----------------------------|--------------------|---|
| A) integer and irrational | <i>integer</i> | ✓ |
| B) rational and real | <i>rational</i> | ✓ |
| C) irrational and rational | <i>real number</i> | ✓ |
| D) real and natural | <i>natural</i> | ✗ |
| | <i>irrational</i> | ✗ |

*So **B** is the correct description.*

a) Use true and false to complete this table:

| | | | | |
|-------|--------------|-------------|--------------|-------------|
| | integer | rational | irrational | real |
| 4.327 | false | true | false | true |

b) Use true and false to complete this table:

| | | | | |
|------|---------|----------|------------|------|
| | integer | rational | irrational | real |
| -500 | | | | |

c) Use true and false to complete this table:

| | | | | |
|-------|---------|----------|------------|------|
| | integer | rational | irrational | real |
| π | | | | |

d) Use true and false to complete this table:

| | | | | |
|----------------|---------|----------|------------|------|
| | integer | rational | irrational | real |
| $\frac{3}{14}$ | | | | |

e) Which classes of numbers describe 0.65291...?

- A) integer and rational
 - B) rational and real
 - C) integer and irrational
 - D) irrational and real
-

f) Which classes of numbers describe $-\sqrt{49}$?

- A) integer and rational
 - B) irrational and real
 - C) integer and irrational
 - D) rational and irrational
-

g) Which classes of numbers describe $0.\bar{1}5384\bar{6}$?

- A) integer and irrational
 - B) irrational and real
 - C) integer and rational
 - D) rational and real
-

h) Which classes of numbers describe $\frac{257}{43}$?

- A) integer and rational
 - B) irrational and real
 - C) rational and real
 - D) rational and irrational
-

- Simplify the radical to the simplest form.

To estimate the value of a radical:

EITHER

- Find the perfect squares greater than ($>$) and less than ($<$) the number inside the square root.
- OR

- Find the rational approximation of the radical from a table of values. (see Math Facts, page 437)
- Express the real numbers as decimals. (see Skill Builder 7.6, page 76)
- Order the decimal numbers. (see Skill Builder 7.1, page 71)

Q. Place in descending order:

$$\sqrt{7}, \frac{7}{3}, 2.3, \frac{10}{4}, 2.41$$

A. $\sqrt{7} = 2.6457\dots$

$$\frac{7}{3} = 2.3333\dots$$

$$\frac{10}{4} = 2.5$$

Descending means from the largest to smallest: 2.6457, 2.5, 2.41, 2.3333, 2.3

The order is: $\sqrt{7}, \frac{10}{4}, 2.41, \frac{7}{3}, 2.3$

a) Which number is greater:

$$\pi \text{ or } 3.1? \quad \left(\pi \approx 3.14159 \right)$$

b) Which number is greater:

$$\pi \text{ or } \sqrt{10}?$$

c) Which number is smaller:

$$\sqrt{20} \text{ or } 5? \quad \left(\sqrt{20} = 2\sqrt{5} \approx 4.4721 \right)$$

d) Which number is smaller:

$$\sqrt{6} \text{ or } 2?$$

e) Which number is smaller:

$$5 \text{ or } \sqrt{32}?$$

f) Which number is greater:

$$\sqrt{50} \text{ or } 7?$$

g) Place in ascending order:

$$\sqrt{6}, \frac{3}{2}, 2.6, 2.\bar{4}$$

h) Place in descending order:

$$\sqrt{5}, \frac{5}{2}, 1.2, \sqrt{2}, 1.5$$

i) Place in descending order:

$$\frac{8}{3}, \sqrt{4}, \sqrt{8}, 2.6, 2.75$$

j) Place in ascending order:

$$3.2, \sqrt{9}, \pi, \frac{9}{4}, \sqrt{7}$$