17. [Algebra - Factorization]

Skill 17.1 Factoring by finding the GCF of the coefficients.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 46)
- Write the GCF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factor $15a - 24$

A. $15a - 24$

$3 \cdot 5a = 15a$ and $3 \cdot 8 = 24$

GCF of 15 and 24 is 3

Remaining factors are 5a and 8.

Write the GCF before the ( ) $= 3(5a - 8)$ Keep the sign

a) Factor $4k - 16$ = $4(k - 4)$

$4 \cdot k = 4k$ and $4 \cdot 4 = 16 \Rightarrow$ GCF is 4

b) Factor $4x + 8$ =

c) Factor $6s + 18$ =

d) Factor $3u - 15$ =

e) Factor $9m - 24$ =

f) Factor $14n + 21$ =

g) Factor $2y + 10z$ =

h) Factor $4a - 12b$ =

i) Factor $6d + 14e$ =

j) Factor $16uv - 40u$ =

k) Factor $3m - 6n + 9$ =

l) Factor $4g + 4h - 6$ =

m) Factor $5h^2 - 10i + 25j$ =

n) Factor $6r^2 - 27s + 9t$ =

Skill 17.2 Factoring by finding the GCF of coefficients and variables.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 46)
- Find any common factors (CF) from the pronumerals in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factor $18kl - 24k$

A. $18kl - 24k$

$6 \cdot 3 = 18$ and $6 \cdot 4 = 24$

GCF of 18 and 24 is 6

k is common to both terms

Remaining factors are $3l$ and 4.

Write all CF's before the ( )

$= \mathbf{6k(3l - 4)}$

Keep the sign

a) Factor $ab + 5b$

b is common to both terms $= b(a + 5)$

$ab = ba$

b) Factor $de + d$

$= $

c) Factor $7e + ef$

$= $

d) Factor $3st + 4s$

$= $

e) Factor $8ab - 4b$

$= $

f) Factor $15g + 20gh$

$= $

g) Factor $wx - xy$

$= $

h) Factor $2jk + 2kl$

$= $

i) Factor $uv - 3vw$

$= $

j) Factor $8ab + 4bc$

$= $

k) Factor $12qr + 8rs$

$= $

l) Factor $15de - 6ef$

$= $

m) Factor $6xy + 9yz$

$= $

n) Factor $10gh - 25gi$

$= $

Skill 17.3 Factoring to simplify expressions involving large numbers.

- Find the factor repeating in both products.
- Write this number in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factor and evaluate
$45 \times 7 + 45 \times 3$

A. $45 \times 7 + 45 \times 3$ (45 is repeating)
$= 45 \times (7 + 3)$
$= 45 \times 10$
$= \mathbf{450}$

a) Factor and evaluate
$99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$
$= 99 \times 1 \quad = \boxed{99}$

b) Factor and evaluate
$15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$
$= \quad = \boxed{}$

c) Factor and evaluate
$987 \times 2 + 987 \times 8$

$=$
$= \quad = \boxed{}$

d) Factor and evaluate
$40 \times 8 + 40 \times 12$

$=$
$= \quad = \boxed{}$

e) Factor and evaluate
$23 \times 37 + 23 \times 63$

$=$
$= \quad = \boxed{}$

f) Factor and evaluate
$25 \times 26 + 25 \times 24$

$=$
$= \quad = \boxed{}$

g) Factor and evaluate
$999 \times 9 - 999 \times 8$

$=$
$= \quad = \boxed{}$

h) Factor and evaluate
$87 \times 19 - 87 \times 9$

$=$
$= \quad = \boxed{}$

i) Factor and evaluate
$-4 \times 14 - 4 \times 6$
(Both terms are negative so CF is negative; $- + = -$)

$= -4 \times (14 + 6)$
$= \quad = \boxed{}$

j) Factor and evaluate
$-9 \times 33 - 9 \times 67$

$=$
$= \quad = \boxed{}$

Skill 17.4 Factoring involving squared terms.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 46)
- Find any common factors (CF) from the variable in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factor $2wx - 12w^2x$

A. $2wx - 12w^2x$

$2 \cdot 1 = 2$ and $2 \cdot 6 = 12$

GCF of 2 and 12 is 2

wx is common to both terms

Remaining factors are 1 and 6w.

Write all CF's before the ( ) $= \mathbf{2wx(1 - 6w)}$ *Keep the sign*

a) Factor $2j^2k + 5j$

$CF = j$ $= j(2jk + 5)$

Write all CF's before the ( )

b) Factor $e^2 + 7e$

..... $=$ .....

c) Factor $h + 4h^2$

..... $=$ .....

d) Factor $m^2 - 9m$

..... $=$ .....

e) Factor $3c - 12c^2$

..... $=$ .....

f) Factor $4f^2 + 6f$

..... $=$ .....

g) Factor $fg^2 + f$

..... $=$ .....

h) Factor $10b - 16ab^2$

..... $=$ .....

i) Factor $14bc + 2b^2c$

..... $=$ .....

j) Factor $5r^2s - r^2t$

..... $=$ .....

k) Factor $vw + 7v^2 - 3vwx$

..... $=$ .....

l) Factor $8j^2 - 24jk + 12jl$

..... $=$ .....

m) Factor $f^3g^2 + fg^2$

..... $=$ .....

n) Factor $p^3q^2 + p^2q + pq$

..... $=$ .....

Skill 17.5 Factoring negative terms.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 46)
- Find any common factors (CF) from the variables in each term.
- Write the GCF and any other CF in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factor $-10r^2 - 5r$

A. $-10r^2 - 5r$
$5 \cdot 2 = 10$ and $5 \cdot 1 = 5$
GCF of 10 and 5 is 5
r is common to both terms
– is common to both terms
Write all CF before the ( )
Remaining factors are r and 1.
$= \mathbf{-5r(r + 1)}$

"–" is common to both terms

a) Factor $-7a - 21$ = $-7(a + 3)$

$7 \cdot 1 = 7$ and $7 \cdot 3 = 21 \Rightarrow$ GCF is 7

b) Factor $-4k - 12$ =

$4 \cdot 1 = 4$ and $4 \cdot 3 = 12 \Rightarrow$ GCF is

c) Factor $-6g - 15$ =

d) Factor $-6e - 14$ =

e) Factor $-2h^2 - 6h$ =

f) Factor $-8z^2 - 28z$ =

g) Factor $-12i^3 - 9ij$ =

h) Factor $-t^3 - 5t^2u$ =

i) Factor $-2x^3 - 4xy$ =

j) Factor $-4m^3 - 12mn^2 + 18m$ =

Skill 17.6 Factoring by finding binomial factors.

- Find any common factors (CF).
  Hint: It might help to think of common factors that are expressions like (d + 2) as a blob ■.
- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factor $2(r-1)-r(r-1)$

A. $2(r-1)-r(r-1)$ — Consider (r – 1) = ■
$= 2\blacksquare - r\blacksquare$ — Keep the sign
$= \blacksquare\,(2-r)$
$= \mathbf{(r-1)(2-r)}$

a) Factor
$d(d+2)+8(d+2)$ — Consider (d + 2) = ■; Keep the sign
$= d\blacksquare + 8\blacksquare$
$= \blacksquare\,(d+8)$ $= \boxed{(d+2)(d+8)}$

b) Factor
$2(h-3)+h(h-3)$
$= 2\blacksquare + h\blacksquare$
$=$ ..... $=$ ☐

c) Factor
$5(x+4)+x(x+4)$
$=$ .....
$=$ ..... $=$ ☐

d) Factor
$b(b-7)+6(b-7)$
$=$ .....
$=$ ..... $=$ ☐

e) Factor
$a(a+2)-9(a+2)$
$=$ .....
$=$ ..... $=$ ☐

f) Factor
$z(z-5)-(z-5)$
$=$ .....
$=$ ..... $=$ ☐

g) Factor
$j^2+4j+j+4$
$=$ .....
$=$ ..... $=$ ☐

h) Factor
$mn-2m+4n-8$
$=$ .....
$=$ ..... $=$ ☐

i) Factor
$qs-3q+st-3t$
$=$ .....
$=$ ..... $=$ ☐

j) Factor
$12vw-6v+8w-4$
$=$ .....
$=$ ..... $=$ ☐

Skill 17.7 Factoring using the difference of perfect squares.

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$(a + b)(a - b) = a \times a + a \times -b + b \times a + b \times -b$

$= a^2 - ab + ba - b^2$

$= a^2 - b^2$

Group like terms

Q. Factor $5w^2 - 20$

A. $5w^2 - 20$

Take out the CF of 5

$= 5(w^2 - 4)$

$4 = 2^2$

$= 5(w^2 - 2^2)$

Use $a^2 - b^2 = (a + b)(a - b)$ where $a = w$ and $b = 2$

$= \mathbf{5(w + 2)(w - 2)}$

a) Factor $c^2 - 81$

$b^2 = 81$, $b = 9$

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2$ $= (c + 9)(c - 9)$

b) Factor $y^2 - 4$

.....

= ..... = ...

c) Factor $d^2 - e^2$

.....

= ..... = ...

d) Factor $36 - h^2$

.....

= ..... = ...

e) Factor $4j^2 - 9$

.....

= ..... = ...

f) Factor $2c^2 - 50$

.....

= ..... = ...

g) Factor $p^2 - 81q^2$

.....

= ..... = ...

h) Factor $80 - 5y^2$

.....

= ..... = ...

i) Factor $3d^2 - 27$

.....

= ..... = ...

j) Factor $100 - 4k^2$

.....

= ..... = ...

Skill 17.8 Factoring four terms by grouping 2 and 2.

- Begin factoring by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factor again by finding the common binomial factor. (see skill 17.6, page 183)
  Hint: It might help to think of binomial factors that are expressions like $(d + 2)$ as a blob ■.
- Take out the binomial factor or blob and write it in front the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

Q. Factor $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$ — Keep the sign

$= m(m + 3) + 5(m + 3)$

$= m\blacksquare + 5\blacksquare$ — Consider $(m + 3) = \blacksquare$

$= \blacksquare(m + 5)$

$= \boldsymbol{(m + 3)(m + 5)}$

a) Factor $c^2 + 8c + 3c + 24$ — Group 2 and 2, Factor each group

$= c(c + 8) + 3(c + 8)$ — Factor again, Consider $(c + 8) = \blacksquare$

$= \blacksquare(c + 3)$ $= (c + 8)(c + 3)$

b) Factor $a^2 + 3a + 2a + 6$

$= a(a + 3) + 2(a + 3)$

$=$ ..... $=$ .....

c) Factor $s^2 + 6s + 5s + 30$

$=$ .....

$=$ ..... $=$ .....

d) Factor $h^2 + 5h + 4h + 20$

$=$ .....

$=$ ..... $=$ .....

e) Factor $v^2 + 7v + 3v + 21$

$=$ .....

$=$ ..... $=$ .....

f) Factor $4n + n^2 + 16 + 4n$

$=$ .....

$=$ ..... $=$ .....

g) Factor $6t + t^2 - 42 - 7t$

$=$ .....

$=$ ..... $=$ .....

h) Factor $4b + 4 - b^2 - b$

$=$ .....

$=$ ..... $=$ .....

Skill 17.9 Factoring quadratic trinomials.

- Write two sets of brackets. Because x^2 can only be produced from $x \cdot x$, write the factors of the squared variable in the brackets $(x\quad)(x\quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factor $x^2 - 9x + 8$

A. $x^2 - 9x + 8$ — Write x in the brackets

$= (x\quad)(x\quad)$ — List the pairs of factors of +8

$8 = 1 \times 8 = -1 \times (-8) = 2 \times 4 = -2 \times (-4)$

Only $-1x$ and $-8x$ can make $-9x$ — Determine the x terms

$= x^2 - 1x - 8x + 8$

$= \boldsymbol{(x - 1)(x - 8)}$

CHECK

$= (x - 1)(x - 8)$ — Expand the brackets

$= x^2 - 1x - 8x + 8$

$= x^2 - 9x + 8$

$= \boldsymbol{(x - 1)(x - 8)}$ ✔

AND/OR consider

$= x^2 - 1x - 8x + 8$ — Group 2 and 2

$= x(x - 1) - 8(x - 1)$ — Factor each group

$= x\blacksquare - 8\blacksquare$

$= \blacksquare(x - 8)$ — Factor again. Consider $(x - 1) = \blacksquare$

$= \boldsymbol{(x - 1)(x - 8)}$

a) Find the missing factor
$x^2 + 7x + 10$

$10 = 2 \cdot 5 = -2 \cdot (-5)$

$5x + 2x = 7x \quad = (x + 5)(x + 2)$

b) Find the missing factor
$d^2 - 4d + 4$

$4 = 2 \cdot 2 = -2 \cdot (-2)$

$= (d - 2)(\quad)$

c) Find the missing factor
$s^2 + 4s + 3$

$= (s + 3)(\quad)$

d) Find the missing factor
$g^2 + 8g + 15$

$= (g + 5)(\quad)$

e) Factor
$m^2 + 2m - 24$

Which pair can be added to get +2m?

$-24 = -4 \cdot 6 = 4 \cdot (-6)$

$6m - 4m = 2m \quad =$

f) Factor
$j^2 + 11j + 24$

$=$

g) Factor
$c^2 - 6c + 5$

$=$

h) Factor
$p^2 - 6p - 16$

$=$