

# 17. [Factorization]

## Skill 17.1 Factoring by finding the GCF of the coefficients.

MMMaive 11 22 33 44  
MMLime 11 22 33 44

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the GCF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.

**Q.** Factor  $15a - 24$

**A.**  $15a - 24$

$3 \cdot 5a = 15a$  and  $3 \cdot 8 = 24$

GCF of 15 and 24 is 3

Remaining factors are 5a and 8.

Write the GCF before the ( ) =  $3(5a - 8)$  Keep the sign

**a)** Factor  $4k - 16$  =

**b)** Factor  $4x + 8$  =

$4 \cdot k = 4k$  and  $4 \cdot 4 = 16 \Rightarrow$  GCF is 4

**c)** Factor  $6s + 18$  =

**d)** Factor  $3u - 15$  =

**e)** Factor  $9m - 24$  =

**f)** Factor  $14n + 21$  =

**g)** Factor  $2y + 10z$  =

**h)** Factor  $4a - 12b$  =

**i)** Factor  $6d + 14e$  =

**j)** Factor  $16uv - 40u$  =

**k)** Factor  $3m - 6n + 9$  =

**l)** Factor  $4g + 4h - 6$  =

**m)** Factor  $5h^2 - 10i + 25j$  =

**n)** Factor  $6r^2 - 27s + 9t$  =

## Skill 17.2 Factoring by finding the GCF of coefficients and variables.

MMMaive 1 2 3 3 4 4  
MMLime 1 1 2 2 3 3 4 4

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

**Q.** Factor  $18kl - 24k$

**A.**  $18kl - 24k$

$$6 \cdot 3 = 18 \text{ and } 6 \cdot 4 = 24$$

GCF of 18 and 24 is 6

$k$  is common to both terms

Remaining factors are  $3l$  and  $4$ .

Write all CF's before the ( )

$$= 6k(3l - 4)$$

Keep the sign

**a)** Factor  $ab + 5b$

$b$  is common to both terms =  $b(a + 5)$

$ab = ba$

**b)** Factor  $de + d$

=

**c)** Factor  $7e + ef$

=

**d)** Factor  $3st + 4s$

=

**e)** Factor  $8ab - 4b$

=

**f)** Factor  $15g + 20gh$

=

**g)** Factor  $wx - xy$

=

**h)** Factor  $2jk + 2kl$

=

**i)** Factor  $uv - 3vw$

=

**j)** Factor  $8ab + 4bc$

=

**k)** Factor  $12qr + 8rs$

=

**l)** Factor  $15de - 6ef$

=

**m)** Factor  $6xy + 9yz$

=

**n)** Factor  $10gh - 25gi$

=

### Skill 17.3 Factoring to simplify expressions involving large numbers.

- Find the factor repeating in both products.
- Write this number in front of the brackets.  
*Hint: When both terms are negative the negative sign is taken out as a common factor.*
- Write the remaining factors inside the brackets.
- Keep the signs.

**Q.** Factor and evaluate  
 $45 \times 7 + 45 \times 3$

**A.**  $45 \times 7 + 45 \times 3$  *45 is repeating*  
 $= 45 \times (7 + 3)$   
 $= 45 \times 10$   
 $= 450$

**a)** Factor and evaluate  
 $99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$   
 $= 99 \times 1 = \boxed{99}$

**b)** Factor and evaluate  
 $15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$   
 $= \dots = \boxed{\phantom{000}}$

**c)** Factor and evaluate  
 $987 \times 2 + 987 \times 8$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**d)** Factor and evaluate  
 $40 \times 8 + 40 \times 12$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**e)** Factor and evaluate  
 $23 \times 37 + 23 \times 63$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**f)** Factor and evaluate  
 $25 \times 26 + 25 \times 24$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**g)** Factor and evaluate  
 $999 \times 9 - 999 \times 8$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**h)** Factor and evaluate  
 $87 \times 19 - 87 \times 9$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

**i)** Factor and evaluate  
 $-4 \times 14 - 4 \times 6$

$= -4 \times (14 + 6)$  *Both terms are negative so CF is negative*  
 $= \dots = \boxed{\phantom{000}}$

**j)** Factor and evaluate  
 $-9 \times 33 - 9 \times 67$

$= \dots$   
 $= \dots = \boxed{\phantom{000}}$

## Skill 17.4 Factoring involving squared terms.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variable in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

**Q.** Factor  $2wx - 12w^2x$

**A.**  $2wx - 12w^2x$

$2 \cdot 1 = 2$  and  $2 \cdot 6 = 12$

GCF of 2 and 12 is 2

$wx$  is common to both terms

Remaining factors are 1 and  $6w$ .

Write all CF's before the ( )

$= 2wx(1 - 6w)$  Keep the sign

**a)** Factor  $2j^2k + 5j$

$CF = j$   $= j(2jk + 5)$

Write all CF's before the ( )

**b)** Factor  $e^2 + 7e$

$=$

**c)** Factor  $h + 4h^2$

$=$

**d)** Factor  $m^2 - 9m$

$=$

**e)** Factor  $3c - 12c^2$

$=$

**f)** Factor  $4f^2 + 6f$

$=$

**g)** Factor  $fg^2 + f$

$=$

**h)** Factor  $10b - 16ab^2$

$=$

**i)** Factor  $14bc + 2b^2c$

$=$

**j)** Factor  $5r^2s - r^2t$

$=$

**k)** Factor  $vw + 7v^2 - 3vwx$

$=$

**l)** Factor  $8j^2 - 24jk + 12jl$

$=$

**m)** Factor  $f^3g^2 + fg^2$

$=$

**n)** Factor  $p^3q^2 + p^2q + pq$

$=$

## Skill 17.5 Factoring negative terms.

MMMaive 11 22 33 44  
MMLime 11 22 33 44

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the GCF and any other CF in front of the brackets.  
*Hint: When both terms are negative the negative sign is taken out as a common factor.*
- Write the remaining factors inside the brackets.
- Check the signs.

**Q.** Factor  $-10r^2 - 5r$

**A.**  $-10r^2 - 5r$

$5 \cdot 2 = 10$  and  $5 \cdot 1 = 5$

GCF of 10 and 5 is 5

$r$  is common to both terms

$-$  is common to both terms

Remaining factors are  $r$  and  $1$ .

$= -5r(r + 1)$

Write all CF before the ( )

"-" is common to both terms

**a)** Factor  $-7a - 21$

$= -7(a + 3)$

$7 \cdot 1 = 7$  and  $7 \cdot 3 = 21 \Rightarrow$  GCF is 7

**b)** Factor  $-4k - 12$

$=$

$4 \cdot 1 = 4$  and  $4 \cdot 3 = 12 \Rightarrow$  GCF is

**c)** Factor  $-6g - 15$

$=$

**d)** Factor  $-6e - 14$

$=$

**e)** Factor  $-2h^2 - 6h$

$=$

**f)** Factor  $-8z^2 - 28z$

$=$

**g)** Factor  $-12i^3 - 9ij$

$=$

**h)** Factor  $-t^3 - 5t^2u$

$=$

**i)** Factor  $-2x^3 - 4xy$

$=$

**j)** Factor  $-4m^3 - 12mn^2 + 18m$

$=$

## Skill 17.6 Factoring by finding binomial factors.

- Find any common factors (CF).

*Hint: It might help to think of common factors that are expressions like  $(d + 2)$  as a blob ■.*

- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

**Q.** Factor  $2(r - 1) - r(r - 1)$

**A.**  $2(r - 1) - r(r - 1)$  Consider  $(r - 1) = \blacksquare$   
 $= 2 \blacksquare - r \blacksquare$  Keep the sign  
 $= \blacksquare (2 - r)$   
 $= (r - 1)(2 - r)$

**a)** Factor  $d(d + 2) + 8(d + 2)$  Consider  $(d + 2) = \blacksquare$   
 $= d \blacksquare + 8 \blacksquare$  Keep the sign  
 $= \blacksquare (d + 8)$  =  $(d + 2)(d + 8)$

**b)** Factor  $2(h - 3) + h(h - 3)$   
 $= 2 \blacksquare + h \blacksquare$   
 $=$  =

**c)** Factor  $5(x + 4) + x(x + 4)$   
 $=$   
 $=$  =

**d)** Factor  $b(b - 7) + 6(b - 7)$   
 $=$   
 $=$  =

**e)** Factor  $a(a + 2) - 9(a + 2)$   
 $=$   
 $=$  =

**f)** Factor  $z(z - 5) - (z - 5)$   
 $=$   
 $=$  =

**g)** Factor  $j^2 + 4j + j + 4$   
 $=$   
 $=$  =

**h)** Factor  $mn - 2m + 4n - 8$   
 $=$   
 $=$  =

**i)** Factor  $qs - 3q + st - 3t$   
 $=$   
 $=$  =

**j)** Factor  $12vw - 6v + 8w - 4$   
 $=$   
 $=$  =

## Skill 17.7 Factoring using the difference of perfect squares.

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$$\begin{aligned}
 (a + b)(a - b) &= a \times a + a \times -b + b \times a + b \times -b \\
 &= a^2 - ab + ba - b^2 \\
 &= a^2 - b^2
 \end{aligned}$$

Group like terms

**Q.** Factor  $5w^2 - 20$

**A.**  $5w^2 - 20$   
 $= 5(w^2 - 4)$   
 $= 5(w^2 - 2^2)$   
 $= 5(w + 2)(w - 2)$

Take out the CF of 5  
 $4 = 2^2$   
 Use  $a^2 - b^2 = (a + b)(a - b)$   
 where  $a = w$  and  $b = 2$

**a)** Factor  $c^2 - 81$

$b^2 = 81$   
 $b = 9$

Use  $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2 = (c + 9)(c - 9)$

**b)** Factor  $y^2 - 4$

$=$

**c)** Factor  $d^2 - e^2$

$=$

**d)** Factor  $36 - h^2$

$=$

**e)** Factor  $4j^2 - 9$

$=$

**f)** Factor  $2c^2 - 50$

$=$

**g)** Factor  $p^2 - 81q^2$

$=$

**h)** Factor  $80 - 5y^2$

$=$

**i)** Factor  $3d^2 - 27$

$=$

**j)** Factor  $100 - 4k^2$

$=$

## Skill 17.8 Factoring four terms by grouping 2 and 2.

MMMaive 11 22 33 44  
MMLime 11 22 33 44

- Begin factoring by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factor again by finding the common binomial factor. (see skill 17.6, page 190)  
*Hint: It might help to think of binomial factors that are expressions like  $(d + 2)$  as a blob ■.*
- Take out the binomial factor or blob and write it in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

**Q.** Factor  $m^2 + 3m + 5m + 15$

**A.**  $m^2 + 3m + 5m + 15$  Keep the sign  
 $= m(m + 3) + 5(m + 3)$   
 $= m \blacksquare + 5 \blacksquare$  Consider  $(m + 3) = \blacksquare$   
 $= \blacksquare(m + 3)$   
 $= (m + 3)(m + 5)$

**a)** Factor  $c^2 + 8c + 3c + 24$   
Group 2 and 2  
Factor each group  
 $= c(c + 8) + 3(c + 8)$   
Factor again  
Consider  $(c + 8) = \blacksquare$   
 $= \blacksquare(c + 8)$   $(c + 8)(c + 3)$

**b)** Factor  $a^2 + 3a + 2a + 6$   
 $= a(a + 3) + 2(a + 3)$   
 $=$    $=$

**c)** Factor  $s^2 + 6s + 5s + 30$   
 $=$    
 $=$    $=$

**d)** Factor  $h^2 + 5h + 4h + 20$   
 $=$    
 $=$    $=$

**e)** Factor  $v^2 + 7v + 3v + 21$   
 $=$    
 $=$    $=$

**f)** Factor  $4n + n^2 + 16 + 4n$   
 $=$    
 $=$    $=$

**g)** Factor  $6t + t^2 - 42 - 7t$   
 $=$    
 $=$    $=$

**h)** Factor  $4b + 4 - b^2 - b$   
 $=$    
 $=$    $=$

## Skill 17.9 Factoring quadratic trinomials.

- Write two sets of brackets. Because  $x^2$  can only be produced from  $x \cdot x$ , write the factors of the squared variable in the brackets  $(x \quad)(x \quad)$ .
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of  $x$  terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

**Q.** Factor  
 $x^2 - 9x + 8$

**A.**  $x^2 - 9x + 8$   
 $= (x \quad)(x \quad)$   
 $8 = 1 \times 8 = -1 \times (-8) = 2 \times 4 = -2 \times (-4)$   
 Only  $-1x$  and  $-8x$  can make  $-9x$   
 $= x^2 - 1x - 8x + 8$   
 $= (x - 1)(x - 8)$

Write  $x$  in the brackets

List the pairs of factors of  $+8$

Determine the  $x$  terms

AND/OR consider

$= x^2 - 1x - 8x + 8$   
 $= x(x - 1) - 8(x - 1)$   
 $= x \blacksquare - 8 \blacksquare$   
 $= \blacksquare(x - 8)$   
 $= (x - 1)(x - 8)$

Group 2 and 2

Factor each group

Factor again  
Consider  $(x - 1) = \blacksquare$

**CHECK**  
 $(x - 1)(x - 8)$   
 $= x^2 - 1x - 8x + 8$   
 $= x^2 - 9x + 8 \quad \checkmark$

Expand the brackets

**a)** Find the missing factor  
 $x^2 + 7x + 10$

$10 = 2 \cdot 5 = -2 \cdot (-5)$

$= 5x + 2x = 7x \quad = (x + 5)(x + 2)$

**b)** Find the missing factor  
 $d^2 - 4d + 4$

$4 = 2 \cdot 2 = -2 \cdot (-2)$

$= \quad = (d - 2)(\quad)$

**c)** Find the missing factor  
 $s^2 + 4s + 3$

$= \quad = (s + 3)(\quad)$

**d)** Find the missing factor  
 $g^2 + 8g + 15$

$= \quad = (g + 5)(\quad)$

**e)** Factor  
 $m^2 + 2m - 24$

Which pair can be added to get  $+2m$ ?

$-24 = -4 \cdot 6 = 4 \cdot (-6)$

$= 6m - 4m = 2m \quad = \quad = \quad$

**f)** Factor  
 $j^2 + 11j + 24$

$= \quad = \quad = \quad$

**g)** Factor  
 $c^2 - 6c + 5$

$= \quad = \quad = \quad$

**h)** Factor  
 $p^2 - 6p - 16$

$= \quad = \quad = \quad$