

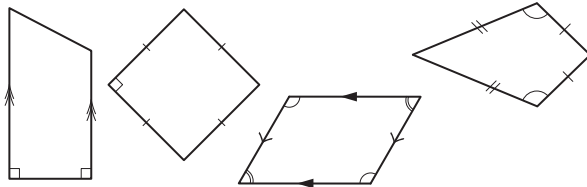
20. [Shapes]

Skill 20.1 Recognizing polygons, quadrilaterals and triangles.

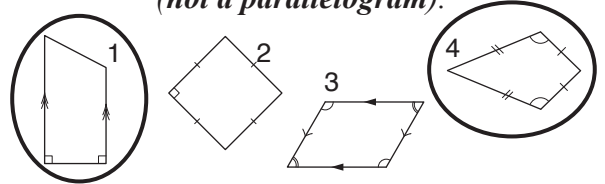
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- Consider the definition of a polygon. (see Glossary, page 422)
- Consider the properties of a parallelogram:
 - both pairs of opposite sides are parallel and equal in length.
- Consider the properties of an isosceles triangle:
 - two sides and two corresponding angles are equal.

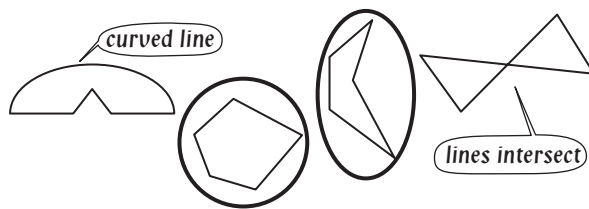
Q. Circle the shapes that are **not** parallelograms.



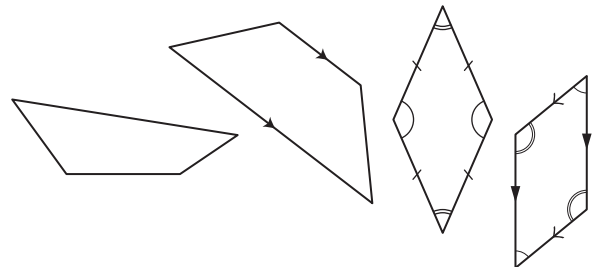
- A.** 1st shape - has only two opposite sides parallel (**not a parallelogram**).
 2nd shape - has both pairs of opposite sides equal in length (parallelogram).
 3rd shape - has both pairs of opposite sides parallel (parallelogram).
 4th shape - doesn't have any parallel sides (**not a parallelogram**).



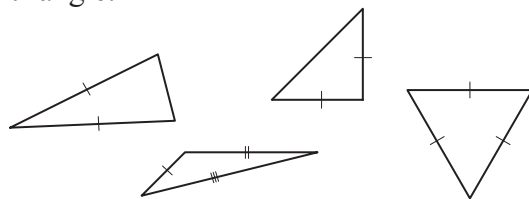
a) Circle the shapes that are polygons.



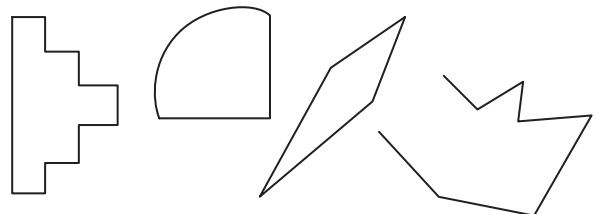
b) Circle the shapes that are **not** parallelograms.



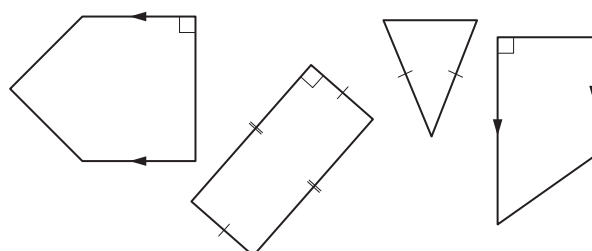
c) Circle the shape that is **not** an isosceles triangle.



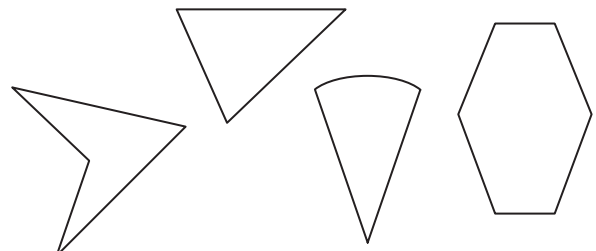
d) Circle the shapes that are **not** polygons.



e) Circle the shape that is a parallelogram.



f) Circle the shape that is **not** a polygon.



Skill 20.2 Classifying triangles.

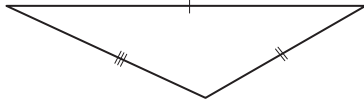
- Look for equal sides or equal angles.
- Look at the types of angles inside the triangle.

| Sides and angles | Triangle type |
|--------------------------|--------------------|
| no equal sides/angles | scalene |
| two equal sides/angles | isosceles |
| three equal sides/angles | equilateral |

| Angles | Triangle type |
|------------------|---------------|
| all acute angles | acute |
| one right angle | right |
| one obtuse angle | obtuse |

Q. Which two options describe this triangle?

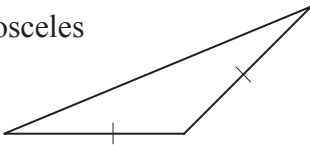
- A) scalene
B) equilateral
C) obtuse



- A.** A) scalene (no equal sides/angles) \Rightarrow true
B) equilateral (all equal sides) \Rightarrow false
C) obtuse (1 obtuse angle) \Rightarrow true
The answer is **A** and **C**.

a) Which two options describe this triangle?

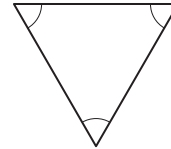
- A) right
B) obtuse
C) isosceles



B and **C**

b) Which two options describe this triangle?

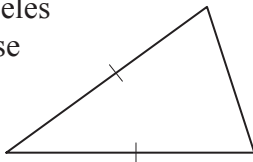
- A) equilateral
B) scalene
C) acute



and

c) Which two options describe this triangle?

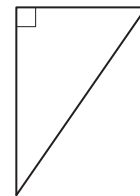
- A) acute
B) isosceles
C) obtuse



and

d) Which two options describe this triangle?

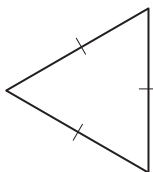
- A) acute
B) right
C) scalene



and

e) Which two options describe this triangle?

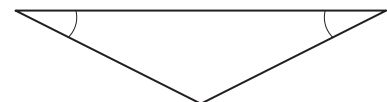
- A) acute
B) scalene
C) equilateral



and

f) Which two options describe this triangle?

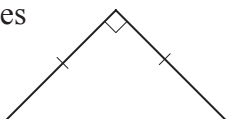
- A) isosceles
B) obtuse
C) right



and

g) Which two options describe this triangle?

- A) acute
B) right
C) isosceles



and

h) Which two options describe this triangle?

- A) scalene
B) isosceles
C) acute



and

Skill 20.3 Describing the properties of quadrilaterals.

- Consider the properties of squares, rectangles, rhombi, parallelograms, kites and trapezoids. (see Glossary)

Q. I am a quadrilateral whose diagonals are not equal in length and bisect each other at right angles. What am I?

- A) square
- B) parallelogram
- C) rhombus
- D) kite

- A.** A) diagonals are equal \Rightarrow A false
 B) diagonals do not bisect each other at right angles \Rightarrow B false
 C) diagonals are not equal and bisect each other at right angles \Rightarrow C true
 D) diagonals don't bisect each other \Rightarrow D false

The answer is **C**.

a) I am a two-dimensional shape with 4 sides. My diagonals are not equal in length and bisect each other but not at right angles. What am I?

- A) rhombus
- B) parallelogram
- C) kite
- D) trapezoid

B

b) I am a quadrilateral with both pairs of opposite sides parallel and diagonals equal in length. What am I?

- A) rhombus
- B) trapezoid
- C) parallelogram
- D) rectangle

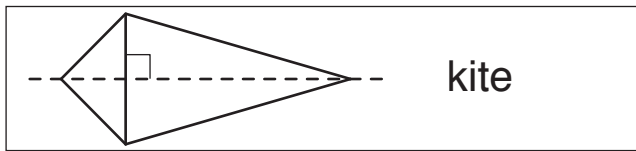
c) I am a two-dimensional shape with 4 sides. Adjacent angles are not equal and I have two axes of symmetry. What am I?

- A) trapezoid
- B) kite
- C) rhombus
- D) rectangle

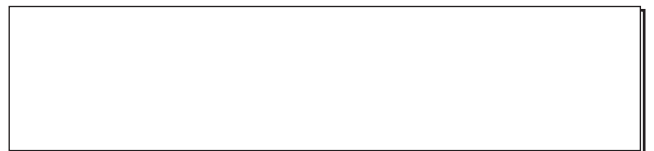
d) I am a quadrilateral with both pairs of opposite sides equal in length, but no axis of symmetry. What am I?

- A) square
- B) trapezoid
- C) parallelogram
- D) rhombus

e) Draw and name the quadrilateral which has diagonals that are perpendicular, but has only one axis of symmetry.



f) Draw and name the quadrilateral which has diagonals that are equal in length and bisect each other at right angles.



g) Draw and name the quadrilateral which has pairs of opposite angles that are equal, but does not have any axis of symmetry.



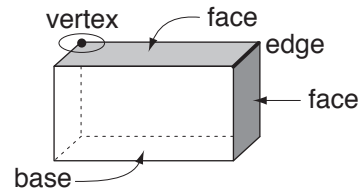
h) Draw and name the quadrilateral which has diagonals that bisect each other at right angles and has two axes of symmetry.



Skill 20.4 Describing the properties of three-dimensional shapes.

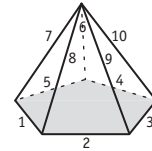
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- Count the number of:
 - faces
 - edges
 - vertices (points/corners)

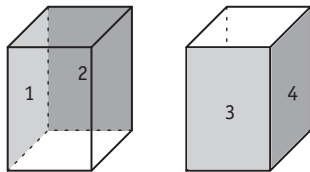


Q. How many edges does a pentagonal pyramid have?

A. Count the number of edges, or straight lines in the pyramid: 5 edges in the base and 5 vertical edges.
The answer is **10**



a) Of the 6 faces of a square prism, how many are rectangles?

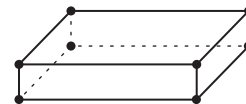


4

b) Of the 7 faces of a pentagonal prism, how many are rectangles?

c) How many vertices are there on a triangular prism?

d) How many vertices are there on a rectangular prism?



e) How many edges are there on a tetrahedron?

f) How many edges are there on a rectangular pyramid?

g) How many faces are there on a pentagonal pyramid?

h) How many faces are there on a triangular prism?

i) Sketch and name the three-dimensional shape that has 6 faces, all of which are squares.

j) Sketch and name the three-dimensional solid that has 4 faces, all of which are triangles.

k) Sketch and name the three-dimensional shape that has 6 faces, five of which are triangles.

l) Sketch and name the three-dimensional solid that has 6 faces, all of which are rectangles.

Euler's formula for any polyhedra:

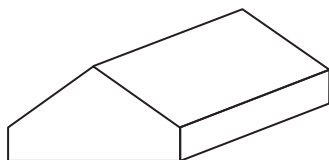
Edges = Vertices + Faces - 2

OR

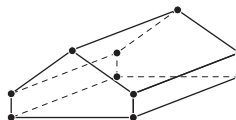
E = V + F - 2

Q. Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this solid:

$$\boxed{} = \boxed{} + \boxed{} - 2$$



A.



$$E = 15$$

$$V = 10$$

$$F = 7$$

$$E = V + F - 2$$

$$15 = 10 + 7 - 2$$

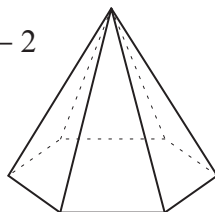
$$15 = 15 \text{ (true)}$$

Substitute 15, 10, 7 into Euler's formula

$$\boxed{15} = \boxed{10} + \boxed{7} - 2$$

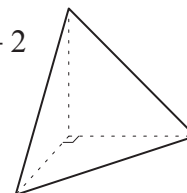
a) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a hexagonal pyramid:

$$\boxed{12} = \boxed{7} + \boxed{7} - 2$$



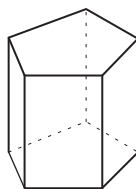
b) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a triangular pyramid:

$$\boxed{} = \boxed{} + \boxed{} - 2$$



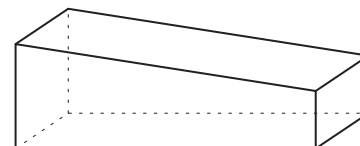
c) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a pentagonal prism:

$$\boxed{} = \boxed{} + \boxed{} - 2$$



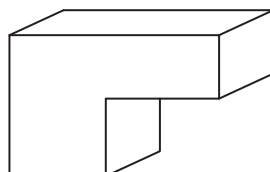
d) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

$$\boxed{} = \boxed{} + \boxed{} - 2$$



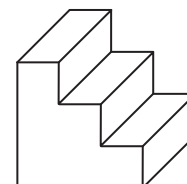
e) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

$$\boxed{} = \boxed{} + \boxed{} - 2$$



f) Euler's formula, $E = V + F - 2$ defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

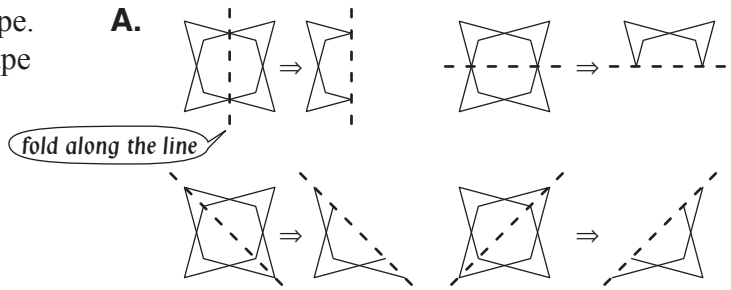
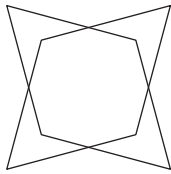
$$\boxed{} = \boxed{} + \boxed{} - 2$$



Skill 20.6 Recognizing line symmetry in two-dimensional shapes.

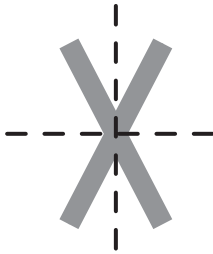
- Imagine along which line the shape can be folded to have one part fit exactly over the other part.

Q. Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



4 axes of symmetry

a) Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



b) Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



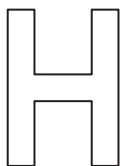
c) Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



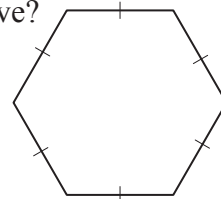
d) Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



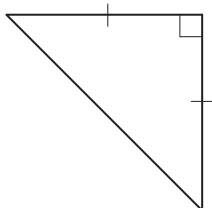
e) Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



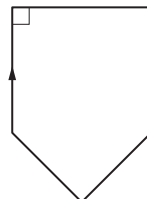
f) Draw all the axes of symmetry for this shape. How many axes of symmetry does this regular hexagon have?



g) Draw and name the shape formed if this triangle is folded along its axis of symmetry.

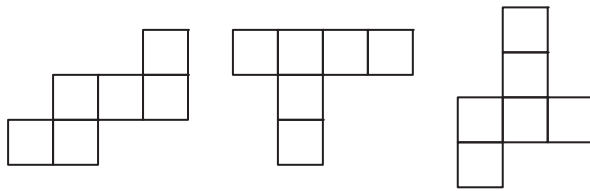


h) Draw and name the shape formed if this shape is folded along its axis of symmetry.



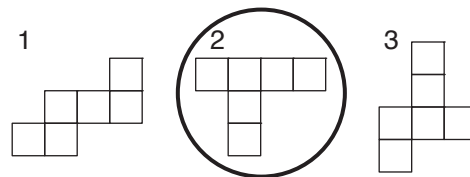
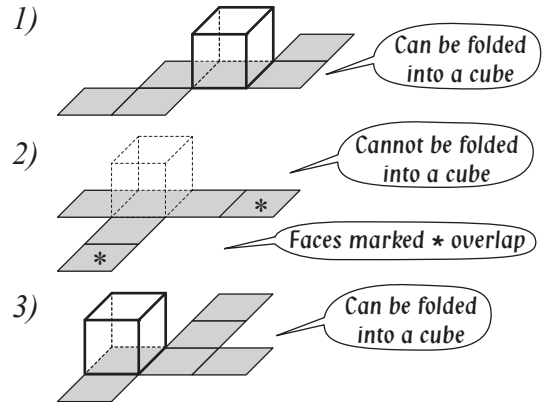
- Identify the shapes in the net. | OR
- Imagine the shape folded. | • Make a model by tracing, cutting out and folding the net.

Q. Circle the net below that **cannot** be folded to make a model of a cube.

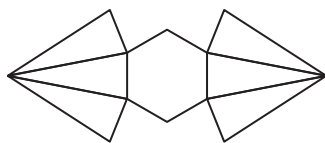


A. Enlarge, trace and cut out the shape, folding to try to form a cube.

OR Imagine the shape folded:

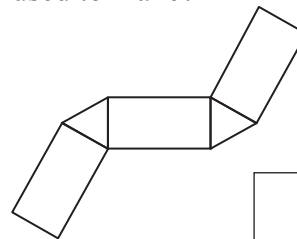


a) What three-dimensional shape can this net be used to make?

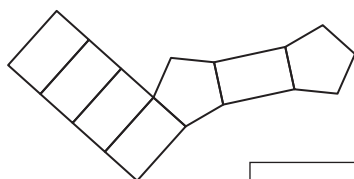


hexagonal pyramid

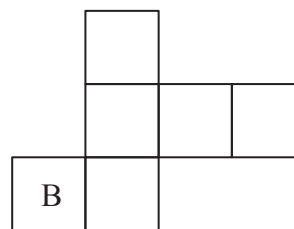
b) What three-dimensional shape can this net be used to make?



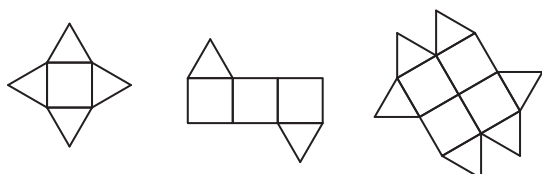
c) What three-dimensional shape can this net be used to make?



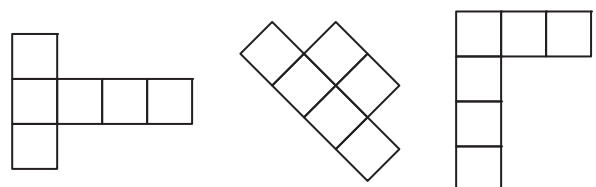
d) On this net of a cube, a face is marked B. Label the opposite face with a T.



e) Circle the net below that **cannot** be folded to make a model of a three-dimensional shape.



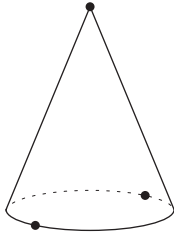
f) Circle the net below that **can** be folded to make a model of a cube.



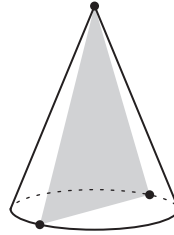
Skill 20.8 Recognizing the shape of cross sections through three-dimensional shapes.

- To name the shape of a cross section through a 3D shape, imagine that you cut the solid through that section, then separate the two parts and look at the shape of the slice.
 - To draw a cross section of a 3D shape through given points, join these points inside the solid.
- Hint: The cross section of a sphere is always a disc, no matter the angle of the cut.*

Q. Draw a cross section through the marked points in this cone. Name the shape drawn.

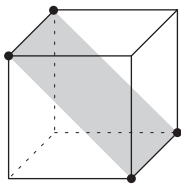


A. Join the three points.



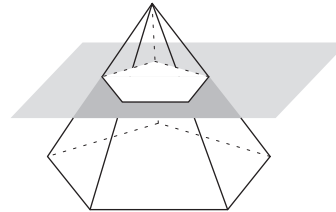
The shape of the cross section is an isosceles triangle.

a) Name the shape of the cross section drawn through this cube.

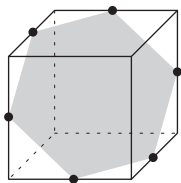


rectangle

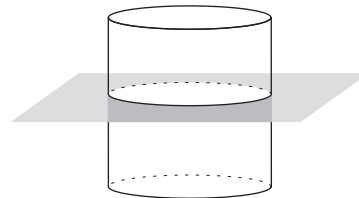
b) Name the shape of the cross section drawn through this pyramid.



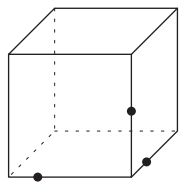
c) Name the shape of the cross section drawn through this cube.



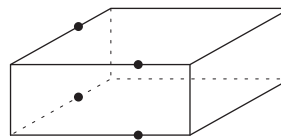
d) Name the shape of the cross section drawn through this cylinder.



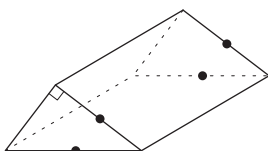
e) Draw a cross section through the marked points in this cube. Name the shape drawn.



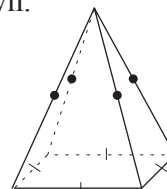
f) Draw a cross section through the marked points in this prism. Name the shape drawn.



g) Draw a cross section through the marked points in this triangular prism. Name the shape drawn.



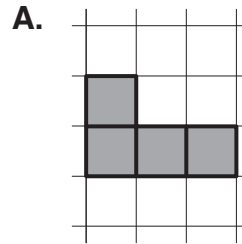
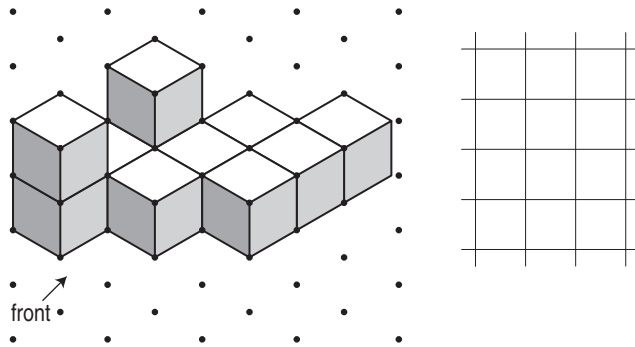
h) Draw a cross section through the marked points in this square pyramid. Name the shape drawn.



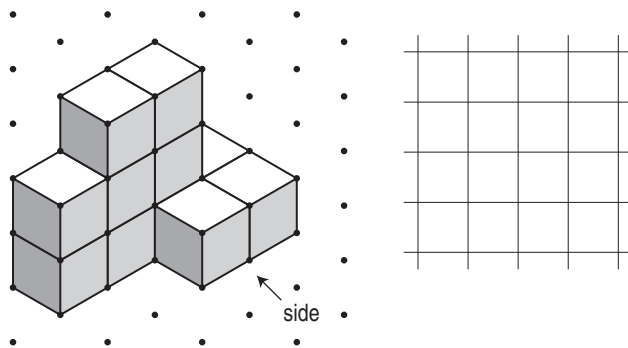
Skill 20.9 Drawing two-dimensional views of three-dimensional shapes.

- To draw the top view of a 3D shape, imagine what you would see if you were looking at the solid from directly above.
- To draw the side view of a 3D shape, imagine what you would see if you were looking at one of the sides of the solid.
- To draw the front view of a 3D shape, imagine what you would see if you were looking at the front of the solid.

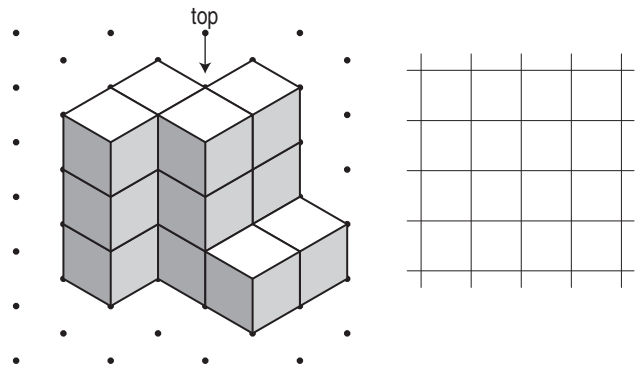
Q. Draw the front view of this solid.



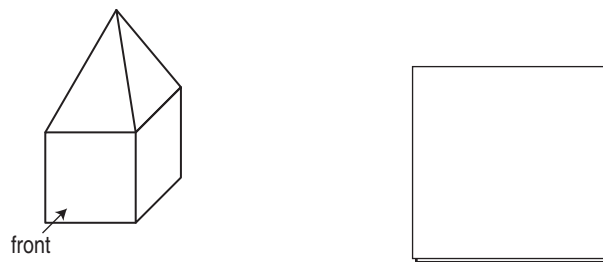
a) Draw the side view of this solid.



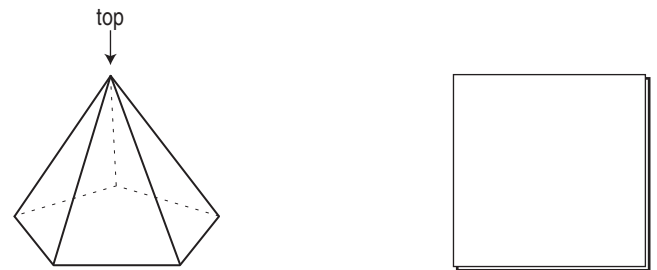
b) Draw the top view of this solid.



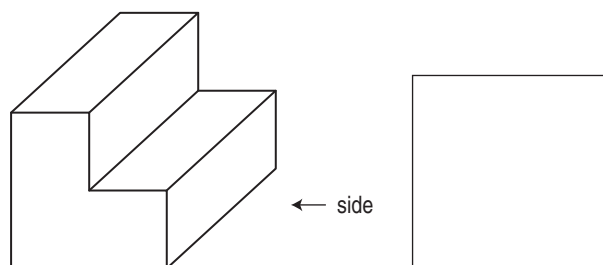
c) Sketch the front view of this solid.



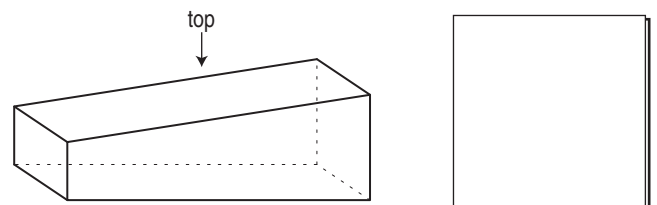
d) Sketch the top view of this solid.



e) Sketch the side view of this solid.



f) Sketch the top view of this solid.



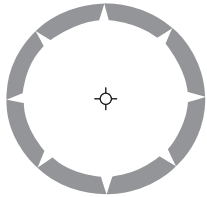
Skill 20.10 Recognizing rotational symmetry in two-dimensional shapes.

- Try to visualize the shape during a full turn of 360° .
- Count how many times during the full turn the image of the shape exactly covers the original shape.
- This number is called the order of rotational symmetry.

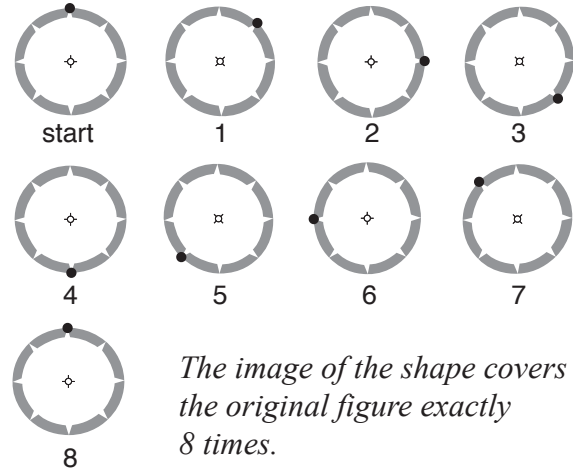
Hints: A shape does not have rotational symmetry if, during a full rotation of 360° , the image of the shape does not exactly cover the original shape.

To count how many times the image of the shape exactly covers the original shape, mark a point on the shape, so you know when the shape has done a complete rotation of 360° .

Q. What is the order of rotational symmetry for this shape?



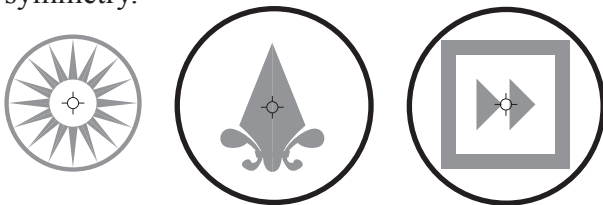
A.



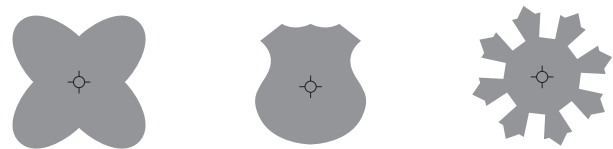
The image of the shape covers the original figure exactly 8 times.

The order of rotational symmetry = 8

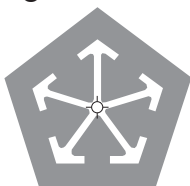
a) Circle the shapes that do not have rotational symmetry.



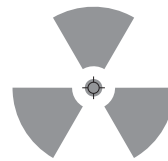
b) Circle the shapes that have rotational symmetry.



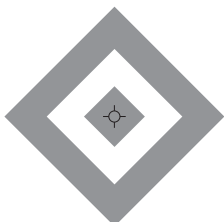
c) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure?



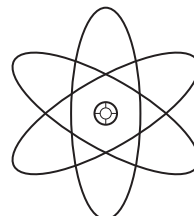
d) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure?



e) What is the order of rotational symmetry for this shape?



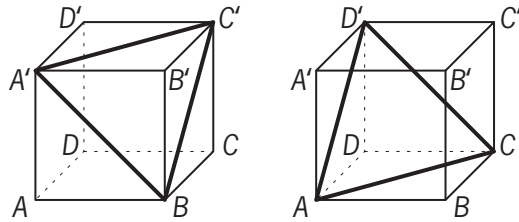
f) What is the order of rotational symmetry for this shape?



Skill 20.11 Finding angle size inside a cube (1).

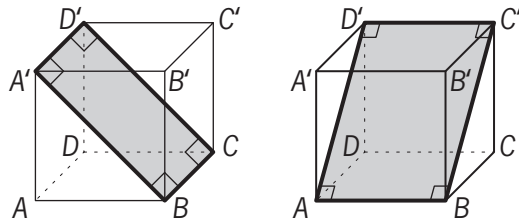
- Consider the definition and the properties of a cube:
 - all faces are squares
 - all edges are equal in length
 - diagonals of all six faces are equal in length.

Example 1:



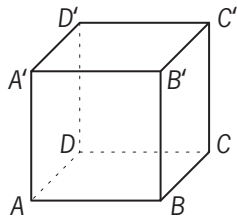
$A'B = BC' = C'A' = AC = CD' = D'A$, so $A'BC'$ and ACD' are equilateral triangles.

Example 2:

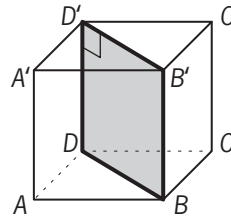


$A'BCD'$ and $ABC'D'$ are rectangles.

Q. How many degrees is $\angle DD'B'$ in this cube?

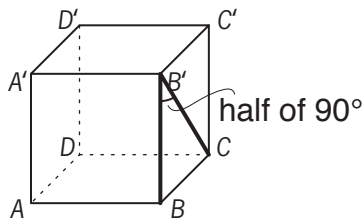


A.

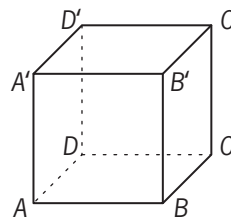


$DBB'D'$ is a rectangle \Rightarrow
 $\angle DD'B'$ is a right angle \Rightarrow
 $\angle DD'B' = 90^\circ$

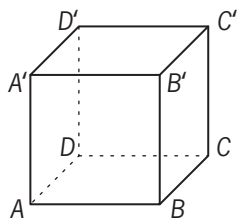
a) How many degrees is $\angle BB'C$ in this cube?



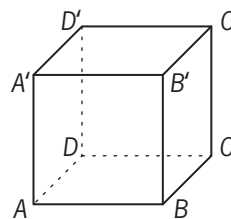
b) How many degrees is $\angle ABC'$ in this cube?



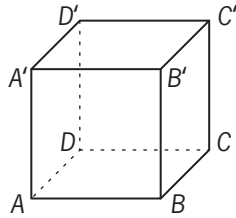
c) How many degrees is $\angle BB'D'$ in this cube?

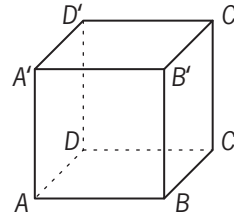


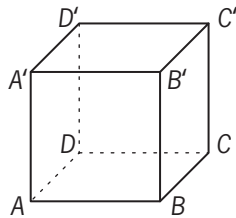
d) How many degrees is $\angle BC'A'$ in this cube?

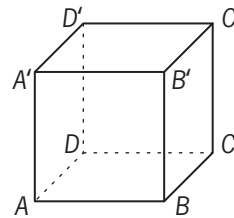


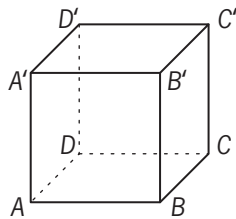
Skill 20.11 Finding angle size inside a cube (2).

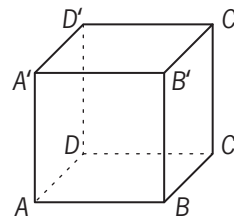
MMMaive 11 22 33 44
MMLime 11 22 33 44e) How many degrees is $\angle DD'A'$ in this cube?

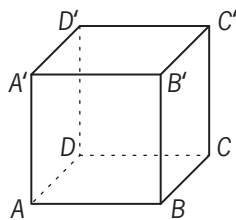
f) How many degrees is $\angle ADC$ in this cube?

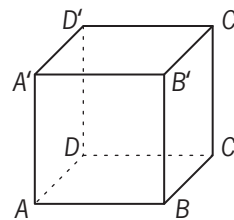
g) How many degrees is $\angle BDC$ in this cube?

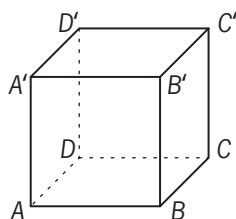
h) How many degrees is $\angle D'A'B$ in this cube?

i) How many degrees is $\angle A'BC'$ in this cube?

j) How many degrees is $\angle C'CD'$ in this cube?

k) How many degrees is $\angle AD'B'$ in this cube?

l) How many degrees is $\angle A'C'D'$ in this cube?

m) How many degrees is $\angle ACC'$ in this cube?

n) How many degrees is $\angle BDD'$ in this cube?