

21. [Angles]

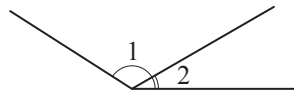
Skill 21.1 Choosing the correct terms related to angles.

MMMaive 1 1 22 33 44
MMLime 11 22 33 44

- Consider the definitions and properties of a variety of angles. (see Glossary or Math Facts, page 444)
*Hints: An angle can be classified according to its size (acute, right, obtuse, straight and reflex).
 Two angles can be classified according to their position in relation to one another (adjacent, supplementary, complementary or vertical).*

Q. Which type of angle describes the pair of angles marked 1 and 2?

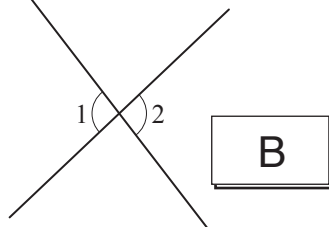
- A) vertical
- B) supplementary
- C) adjacent



- A.** A) *vertical* \Rightarrow angles are congruent
 (1 and 2 are not equal) false
 B) *supplementary* \Rightarrow angles add to 180°
 (1 and 2 add to less than 180°) false
 C) *adjacent* \Rightarrow angles share the vertex
 and an arm true
 The answer is **C**

a) Which type of angle describes the pair of angles marked 1 and 2?

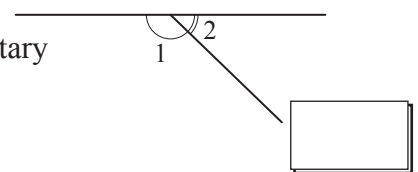
- A) right
- B) vertical
- C) supplementary



B

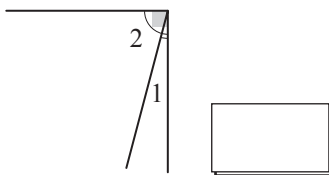
b) Which type of angle describes the pair of angles marked 1 and 2?

- A) straight
- B) supplementary
- C) acute



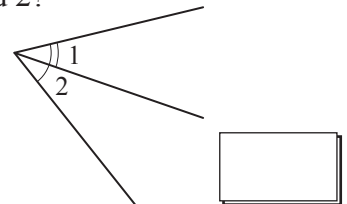
c) Which type of angle describes the pair of angles marked 1 and 2?

- A) reflex
- B) right
- C) complementary



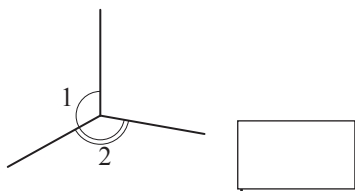
d) Which type of angle describes the pair of angles marked 1 and 2?

- A) acute
- B) obtuse
- C) complementary



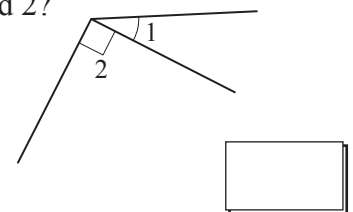
e) Which type of angle describes the pair of angles marked 1 and 2?

- A) supplementary
- B) obtuse
- C) vertical



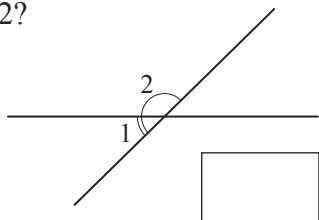
f) Which type of angle describes the pair of angles marked 1 and 2?

- A) complementary
- B) right
- C) adjacent



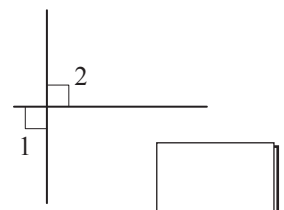
g) Which type of angle describes the pair of angles marked 1 and 2?

- A) supplementary
- B) acute
- C) vertical



h) Which type of angle describes the pair of angles marked 1 and 2?

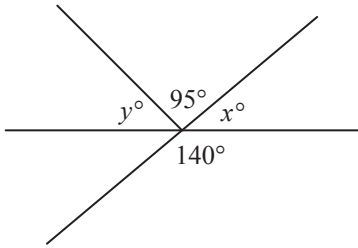
- A) straight
- B) complementary
- C) right



Skill 21.2 Finding the complement and the supplement of a given angle.

- Use the properties:
 - the sum of two complementary angles is 90° .
 - the sum of two supplementary angles is 180° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the values of x° and y° .



A. x° and 140° are supplementary:

$$x^\circ + 140^\circ = 180^\circ$$

$$x^\circ + 140^\circ - 140^\circ = 180^\circ - 140^\circ$$

$$x^\circ = 40^\circ$$

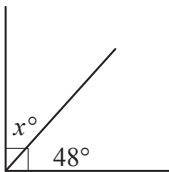
y° , 95° and x° are supplementary:

$$y^\circ + 95^\circ + 40^\circ = 180^\circ$$

$$y^\circ + 135^\circ - 135^\circ = 180^\circ - 135^\circ$$

$$y^\circ = 45^\circ$$

a) Find the value of x° .

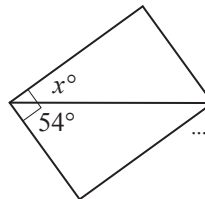


$$x^\circ + 48^\circ = 90^\circ$$

$$x^\circ + 48^\circ - 48^\circ = 90^\circ - 48^\circ$$

$$x^\circ = \boxed{42^\circ}$$

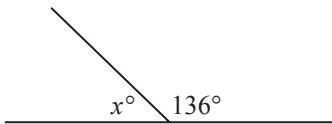
b) Find the value of x° .



$$x^\circ + 54^\circ = 90^\circ$$

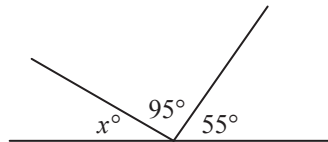
$$x^\circ = \boxed{}$$

c) Find the value of x° .



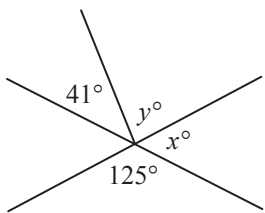
$$x^\circ = \boxed{}$$

d) Find the value of x° .



$$x^\circ = \boxed{}$$

e) Find the values of x° and y° .

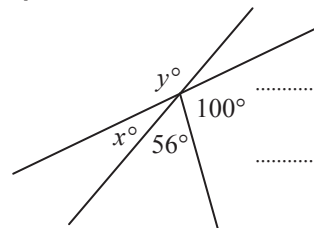


$$x^\circ + 125^\circ = 180^\circ$$

$$x^\circ = \boxed{}$$

$$y^\circ = \boxed{}$$

f) Find the values of x° and y° .



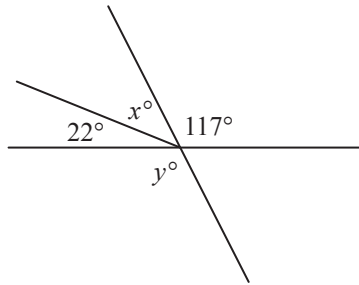
$$x^\circ + 156^\circ = 180^\circ$$

$$x^\circ = \boxed{}$$

$$y^\circ = \boxed{}$$

- Use the definition of vertical angles. (see Glossary, page 444)
- Consider complementary and supplementary angles. (see skill 21.2, page 238)

Q. Find the values of x° and y° .



A. y° and 117° are vertical angles:

$$y^\circ = 117^\circ$$

x° , 22° and 117° are supplementary:

$$x^\circ + 22^\circ + 117^\circ = 180^\circ$$

$$x^\circ + 139^\circ - 139^\circ = 180^\circ - 139^\circ$$

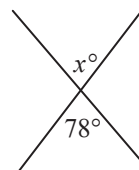
$$x^\circ = 41^\circ$$

a) Find the value of x° .

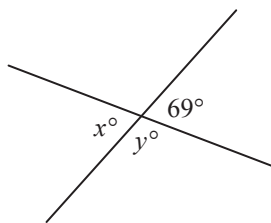


$$26^\circ$$

b) Find the value of x° .



c) Find the values of x° and y° .



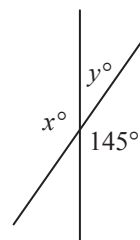
$$x^\circ = \text{[]}$$

$$y^\circ + 69^\circ = 180^\circ$$

$$y^\circ + 69^\circ - 69^\circ = 180^\circ - 69^\circ$$

$$y^\circ = \text{[]}$$

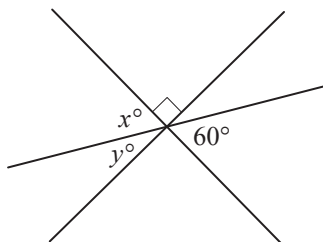
d) Find the values of x° and y° .



$$x^\circ = \text{[]}$$

$$y^\circ = \text{[]}$$

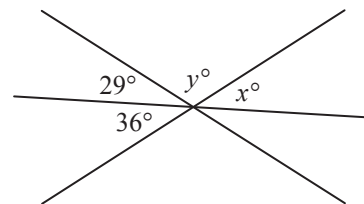
e) Find the values of x° and y° .



$$x^\circ = \text{[]}$$

$$y^\circ = \text{[]}$$

f) Find the values of x° and y° .

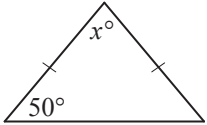


$$x^\circ = \text{[]}$$

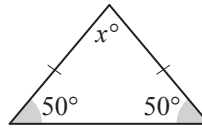
$$y^\circ = \text{[]}$$

- Use the property:
 - the sum of the interior angles of any triangle is 180° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.



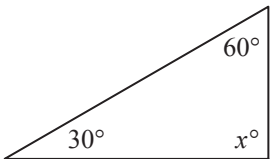
Isosceles triangle \Rightarrow base angles are equal

$$x^\circ + 50^\circ + 50^\circ = 180^\circ$$

$$x^\circ + 100^\circ - 100^\circ = 180^\circ - 100^\circ$$

$$x^\circ = \mathbf{80^\circ}$$

a) Find the value of x° .

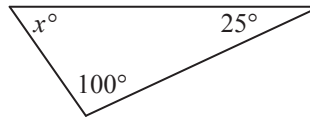


$$x^\circ + 30^\circ + 60^\circ = 180^\circ$$

$$x^\circ + 90^\circ - 90^\circ = 180^\circ - 90^\circ$$

$$x^\circ = \boxed{}$$

b) Find the value of x° .



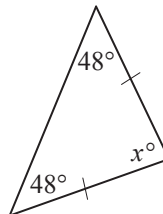
$$x^\circ = \boxed{}$$

c) Find the value of x° .



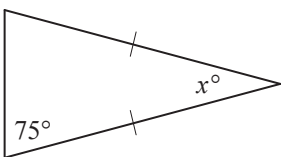
$$x^\circ = \boxed{}$$

d) Find the value of x° .



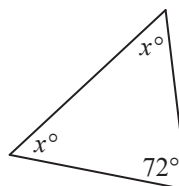
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

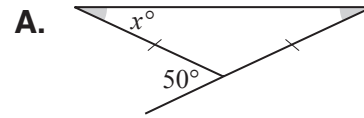
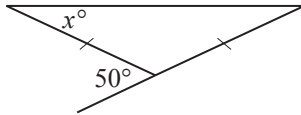
f) Find the value of x° .



$$x^\circ = \boxed{}$$

- Use the property:
 - an exterior angle to a triangle is equal to the sum of the two opposite interior angles of the triangle.
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



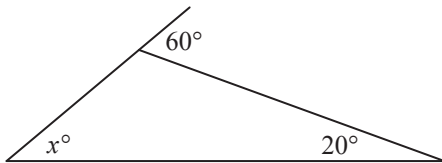
Isosceles triangle \Rightarrow base angles are equal

$$x^\circ + x^\circ = 50^\circ$$

$$2x^\circ \div 2 = 50^\circ \div 2$$

$$x^\circ = 25^\circ$$

a) Find the value of x° .



$$x^\circ + 20^\circ = 60^\circ$$

$$x^\circ + 20^\circ - 20^\circ = 60^\circ - 20^\circ$$

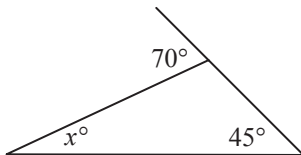
$$x^\circ = \boxed{}$$

b) Find the value of x° .



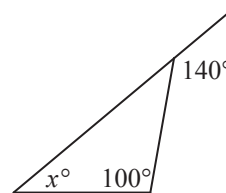
$$x^\circ = \boxed{}$$

c) Find the value of x° .



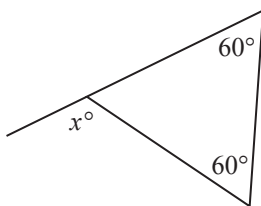
$$x^\circ = \boxed{}$$

d) Find the value of x° .



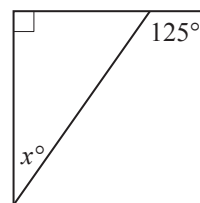
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

f) Find the value of x° .



$$x^\circ = \boxed{}$$

- Use the property:
 - the sum of the interior angles of any quadrilateral is 360° .
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.



Parallelogram \Rightarrow opposite angles are equal

$$2x^\circ + 2 \times 115^\circ = 360^\circ$$

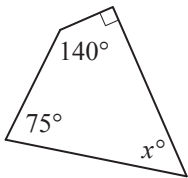
$$2x^\circ + 230^\circ - 230^\circ = 360^\circ - 230^\circ$$

$$2x^\circ = 130^\circ$$

$$2x^\circ \div 2 = 130^\circ \div 2$$

$$x^\circ = 65^\circ$$

a) Find the value of x° .

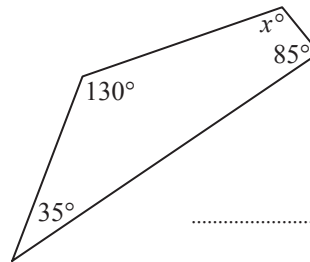


$$x^\circ + 90^\circ + 140^\circ + 75^\circ = 360^\circ$$

$$x^\circ + 305^\circ - 305^\circ = 360^\circ - 305^\circ$$

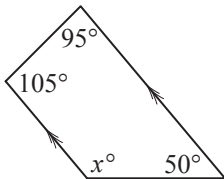
$$x^\circ = \boxed{}$$

b) Find the value of x° .



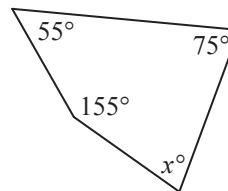
$$x^\circ = \boxed{}$$

c) Find the value of x° .



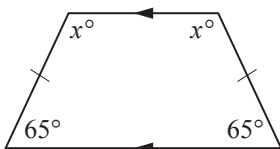
$$x^\circ = \boxed{}$$

d) Find the value of x° .



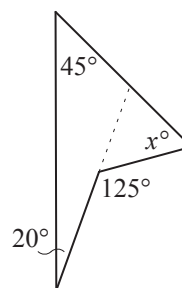
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

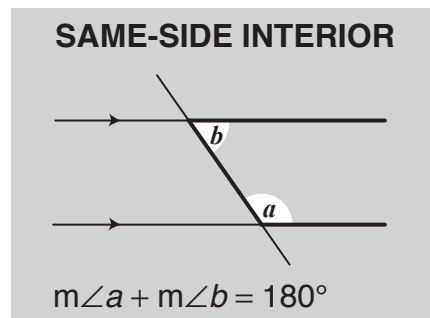
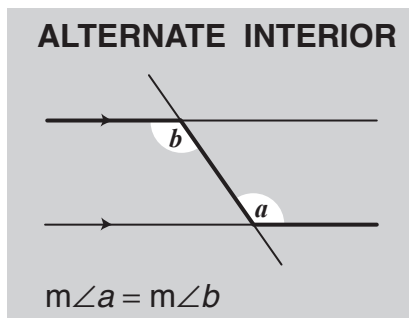
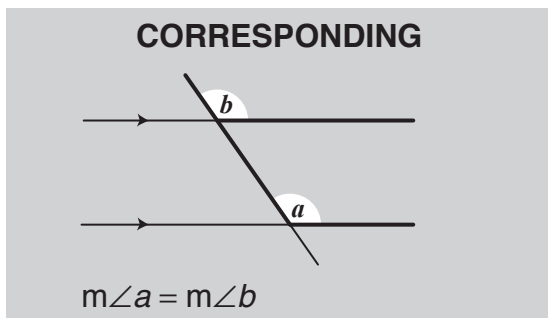
f) Find the value of x° .



$$x^\circ = \boxed{}$$

Skill 21.7 Working with pairs of alternate interior, same-side interior and corresponding angles.

- Consider the classification and properties of the angles formed by intersecting a pair of parallel lines by a transversal. (see Math Facts, page 444)

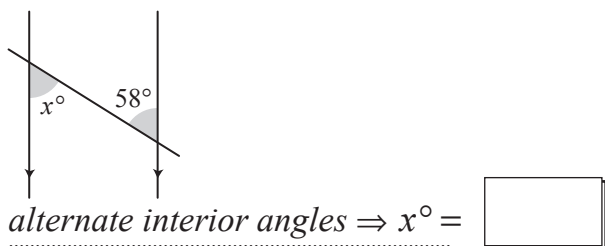


Q. Find the value of x° .

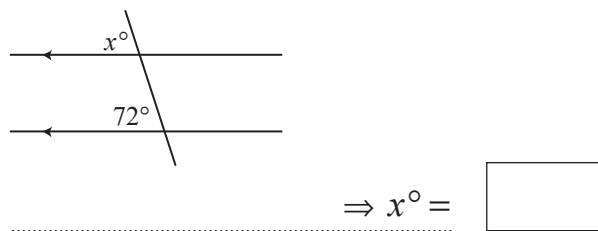
A.
 x° and a° are not in any category above

a° and 134° are corresponding angles
 $\Rightarrow a^\circ = 134^\circ$
 x° and a° are supplementary angles
 $\Rightarrow x^\circ + a^\circ = 180^\circ$
 Substitute $a^\circ = 134^\circ \Rightarrow x^\circ + 134^\circ = 180^\circ$
 $x^\circ + 134^\circ - 134^\circ = 180^\circ - 134^\circ$
 $x^\circ = 46^\circ$

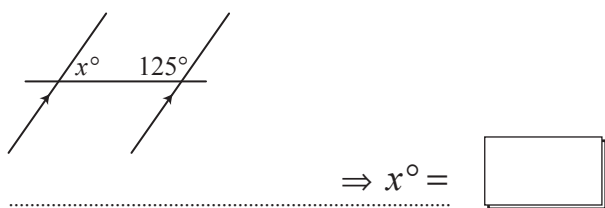
a) Find the value of x° .



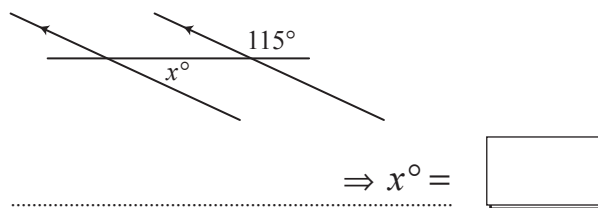
b) Find the value of x° .



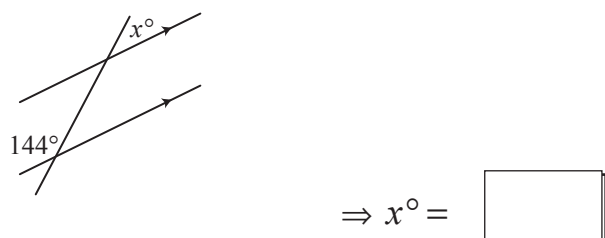
c) Find the value of x° .



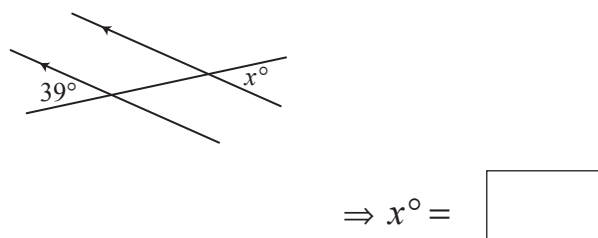
d) Find the value of x° .



e) Find the value of x° .



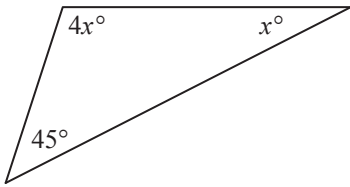
f) Find the value of x° .



Skill 21.8 Finding the value of an angle in a variety of diagrams.

- Use the properties of angles. (see skills 21.1 to 21.7, pages 237 to 243 and Math Facts, page 444)
- Write an equation involving the unknown angle x° .
- Solve the equation for x° .

Q. Find the value of x° .



A.

$$4x^\circ + x^\circ + 45^\circ = 180^\circ$$

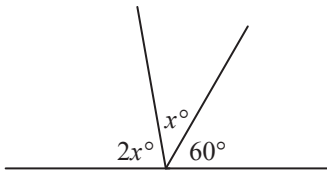
$$5x^\circ + 45^\circ - 45^\circ = 180^\circ - 45^\circ$$

$$5x^\circ = 135^\circ$$

$$5x^\circ \div 5^\circ = 135^\circ \div 5^\circ$$

$$x^\circ = 27^\circ$$

a) Find the value of x° .

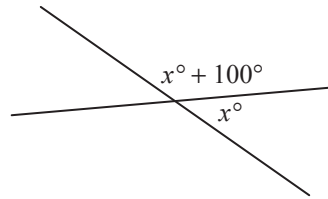


$$3x^\circ + 60^\circ - 60^\circ = 180^\circ - 60^\circ$$

$$3x^\circ \div 3 = 120^\circ \div 3$$

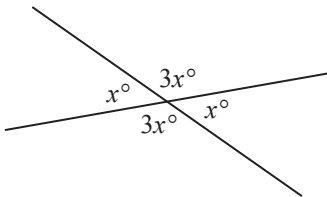
$$x^\circ = \boxed{}$$

b) Find the value of x° .



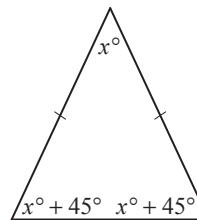
$$x^\circ = \boxed{}$$

c) Find the value of x° .



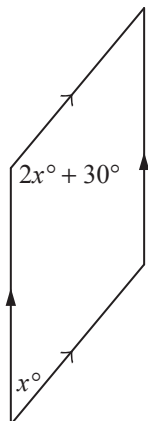
$$x^\circ = \boxed{}$$

d) Find the value of x° .



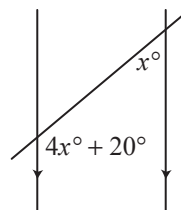
$$x^\circ = \boxed{}$$

e) Find the value of x° .



$$x^\circ = \boxed{}$$

f) Find the value of x° .

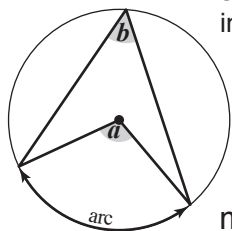


$$x^\circ = \boxed{}$$

Skill 21.9 Finding the value of an angle in a circle (1).

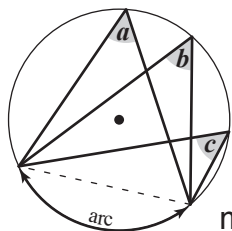
- Use the following properties of angles in circles:

Property 1 The central arc of a circle is twice the size of the inscribed angle which intercepts the same arc of the circle.



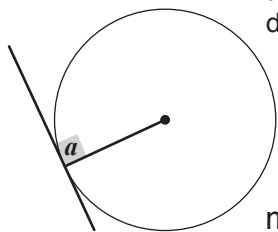
$$m\angle a = 2 \times m\angle b$$

Property 3 All inscribed angles that intercept the same arc of the circle are equal.



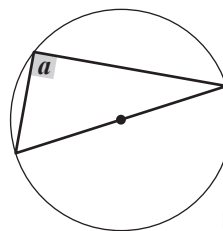
$$m\angle a = m\angle b = m\angle c$$

Property 5 If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.



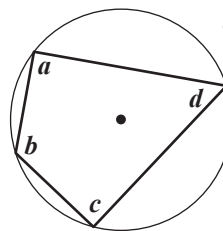
$$m\angle a = 90^\circ$$

Property 2 An angle inscribed in a semicircle is a right angle.



$$m\angle a = 90^\circ$$

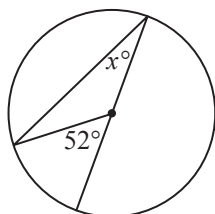
Property 4 The opposite angles in a quadrilateral inscribed in a circle add up to 180° (are supplementary).



$$m\angle a + m\angle c = 180^\circ$$

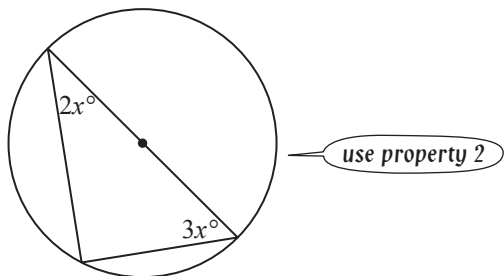
$$m\angle b + m\angle d = 180^\circ$$

Q. Find the value of x° .



A. *52° is a central angle intercepting arc A*
x° is an inscribed angle intercepting arc A $\Rightarrow 2x^\circ = 52^\circ$
 Solve the equation:
 $2x^\circ = 52^\circ$
 $2x^\circ \div 2 = 52^\circ \div 2$
 $x^\circ = 26^\circ$

a) Find the value of x° .

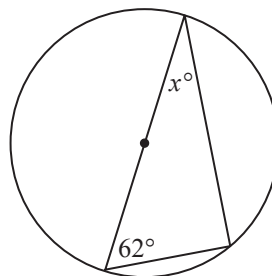


$$2x^\circ + 3x^\circ = 90^\circ$$

$$5x^\circ \div 5 = 90^\circ \div 5$$

$$x^\circ = \boxed{}$$

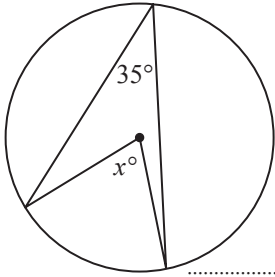
b) Find the value of x° .



$$x^\circ = \boxed{}$$

Skill 21.9 Finding the value of an angle in a circle (2).

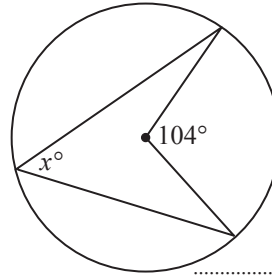
c) Find the value of x° .



$$x^\circ = 2 \times 35^\circ$$

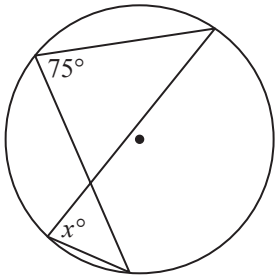
$$x^\circ = \boxed{}$$

d) Find the value of x° .



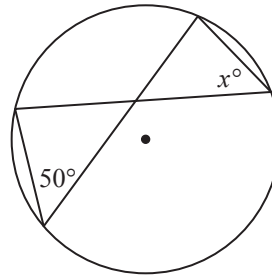
$$x^\circ = \boxed{}$$

e) Find the value of x° .



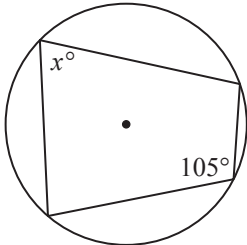
$$x^\circ = \boxed{}$$

f) Find the value of x° .



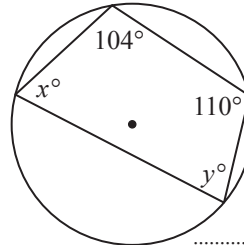
$$x^\circ = \boxed{}$$

g) Find the value of x° .



$$x^\circ = \boxed{}$$

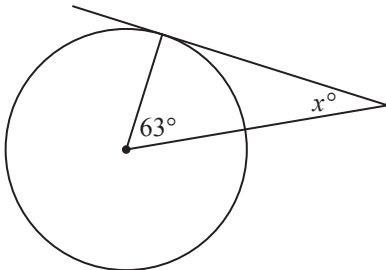
h) Find the values of x° and y° .



$$x^\circ = \boxed{}$$

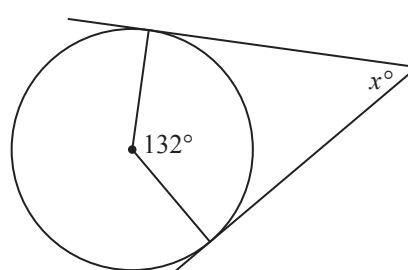
$$y^\circ = \boxed{}$$

i) Find the value of x° .



$$x^\circ = \boxed{}$$

j) Find the value of x° .



$$x^\circ = \boxed{}$$