

9. [Rates / Ratios]

Skill 9.1 Simplifying ratios.

MMMaive 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Write the quantities of the ratio with the same unit of measurement.

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Greatest Common Factor).
- Divide each quantity by the GCF.

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Hints: The order of the quantities in a ratio matters.

' : ' means 'to'.

Examples: The ratio of legs to ears in a dog is $4 : 2 = 2 : 1$

The ratio of ears to legs in a dog is $2 : 4 = 1 : 2$

$$a : b = \frac{a}{b} \quad \text{Ratio}$$

Q. Simplify the ratio
 $80 \text{ min} : 3 \text{ h}$

A. $3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$

$$80 \text{ min} : 3 \text{ h}$$

$$= \overset{4}{80} \text{ min} : \overset{9}{180} \text{ min}$$

$$= 4 : 9$$

Ignore the units

$$1 \text{ h} = 60 \text{ min}$$

GCF of 80 and 180
is 20 so $\div 20$

OR A. $3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$

$$80 \text{ min} : 3 \text{ h}$$

$$= 80 \text{ min} : 180 \text{ min}$$

$$= \overset{4}{8} : \overset{9}{18}$$

$$= 4 : 9$$

Simplify: $\div 10$

Simplify: $\div 2$

a) Simplify the ratio $60c : \$0.20$

$$\$1 = 100c$$

$$\$0.20 = 0.\overset{2}{2}00 \times 100c = 20c$$

2 zeros, 2 places right

$$60c : 20c$$

Simplify: $\div 10$

$$= \overset{3}{6} : \overset{1}{2}$$

Simplify: $\div 2$

$$= \boxed{3 : 1}$$

b) Simplify the ratio $2 \text{ ft} : 6 \text{ in.}$

$$1 \text{ ft} = 12 \text{ in.}$$

$$2 \text{ ft} = 2 \times 12 \text{ in.} = 24 \text{ in.}$$

$$24 \text{ in.} : 6 \text{ in.}$$

Simplify: $\div 6$

$$= 4 : 1$$

$$= \boxed{}$$

c) Simplify the ratio $750 \text{ g} : 1 \text{ kg}$

$$=$$

$$=$$

$$= \boxed{}$$

d) Simplify the ratio $6 \text{ months} : 4 \text{ years}$

$$=$$

$$=$$

$$= \boxed{}$$

e) Simplify the ratio $5 \text{ ft} : 2 \text{ yd} : 8 \text{ in.}$

$$=$$

$$=$$

$$= \boxed{}$$

f) Simplify the ratio $50c : \$4.00 : \2.50

$$=$$

$$=$$

$$= \boxed{}$$

Skill 9.2 Finding the ratio of two or more quantities (1).

- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

Find the largest number that divides evenly into each quantity of the ratio (Greatest Common Factor) and divide each quantity by the GCF.

Hint: The order of the quantities in a ratio matters.

OR

Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Q. Land represents 30% of the earth’s surface and the rest is ocean water. Find the ratio of land to water.

A. $water = 100\% - 70\% = 30\%$
 $land : water$
 $= 30\% : 70\%$ *Simplify: ÷ 10*
 $= 3 : 7$ *Ignore the % sign*

a) An orchestra has 60 strings, 12 brass and 9 woodwinds instruments. What is the ratio of strings to brass to woodwinds instruments?

strings : brass: woodwind

$60 : 12 : 9$

Simplify: ÷ 3 $\overset{20}{60} : \overset{4}{12} : \overset{3}{9} = \boxed{20 : 4 : 3}$

b) Tin foil is made up of 88% tin, 4% copper and the rest lead. What is the ratio of tin to copper to lead in the foil?

$:$ $:$
 $:$ $:$

$= \boxed{}$

c) Find the ratio of the average weight of a Blue Whale (120 ton) to the average weight of a Humpback Whale (30 ton).

$:$
 $:$

$= \boxed{}$

d) Find the ratio of the Boeing 747 wingspan (212 ft) to the Airbus A380 wingspan (260 ft).

$:$
 $:$

$= \boxed{}$

e) A ticket to the musical “Hairspray” on Broadway starts at \$80. A movie ticket to the Village East Cinemas is \$12. What is the ratio of musical to movie ticket prices?

$:$
 $:$

$= \boxed{}$

f) A “Two Day Park Hopper” to Disneyland, Florida, costs \$96 for a child and \$116 for an adult. Find the ratio of adult to child ticket prices?

$:$
 $:$

$= \boxed{}$

Skill 9.2 Finding the ratio of two or more quantities (2).

g) Commercial butter is approximately 80% milk fats and 20% other components. What is the ratio of milk fats to other components?

:

:

=

h) In 2008, of the 100 seats in the Senate, 16 are held by women. What is the ratio of women to men in the Senate?

:

:

=

i) The US House of Representatives was fixed at 435 voting members in 2008. If the Senate has 100 seats, what is the ratio of House seats to Senate seats?

House seats : Senate seats
435 : 100

Simplify: ÷ 5 $\frac{87}{435} : \frac{20}{100} = 87:20$

j) Land represents 30% of the earth's surface. Find the ratio of land to earth's surface.

:

:

=

k) What is the ratio of carbon to total number of atoms in the ethane formula C_2H_6 ?

:

:

=

l) What is the ratio of hydrogen to total number of atoms in the ethylene formula C_2H_4 ?

:

:

=

m) A viscose/polyester blouse has 44% polyester. What is the ratio of viscose to total composition?

:

:

=

n) Of the 2 L of lemonade, 250 mL is concentrated lemon and the rest is soda water. Find the ratio of concentrated lemon to all the lemonade.

:

:

=

o) Of the \$500,000 paid for the property, \$150,000 was for the block of land, and the rest was for building the house. Find the ratio of land to total property price.

:

:

=

p) In his career, Pete Sampras won 14 Grand Slam titles in 52 finals. What is the ratio of win to loss finals?

:

:

=

Skill 9.3 Deciding if two ratios form a proportion.

- Write the two ratios as equal fractions side by side.
- Cross multiply the numerators and the denominators of the fractions.
- If the two products are equal, then the two ratios are in proportion.

A proportion

$$a : b = c : d \quad \text{2 ratios}$$

$$\frac{a}{b} \times \frac{c}{d}$$

$$ad = bc$$

Q. 5 : 2 is in proportion with 25 : 10
True or false?

A. $\frac{5}{2} \times \frac{25}{10}$ *Cross multiply*

$$5 \times 10 = 2 \times 25$$

$$50 = 50$$

true

a) 5 : 6 is in proportion with 3 : 5
True or false?

$$\frac{5}{6} \times \frac{3}{5}$$

$$5 \times 5 = 6 \times 3$$

$$25 = 18$$

false

b) 8 : 12 is in proportion with 6 : 8
True or false?

c) 2 : 12 is in proportion with 3 : 18
True or false?

d) 9 : 15 is in proportion with 30 : 50
True or false?

e) $\frac{12}{15}$ is in proportion with $\frac{3}{5}$
True or false?

$$\frac{12}{15} \times \frac{3}{5}$$

$$12 \times 5 = 15 \times 3$$

$$60 = 45$$

g) $\frac{15}{50}$ is in proportion with $\frac{6}{20}$
True or false?

f) $\frac{8}{20}$ is in proportion with $\frac{20}{50}$
True or false?

h) $\frac{8}{10}$ is in proportion with $\frac{20}{25}$
True or false?

Skill 9.4 Finding the missing term in a proportion.

- Write the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find the missing number (x).

A proportion

$a:b = c:d$ — 2 ratios

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Q. Complete the missing term in the proportion:

10 is to = 5 is to 25

A. $\frac{10}{x} = \frac{5}{25}$

$\frac{10}{x} = \frac{5}{25}$ — Cross multiply

$10 \cdot 25 = x \cdot 5$

$5x = 250$

$\frac{5x}{5} = \frac{250}{5}$ — Simplify: $\div 5$

$x = 50$

a) Complete the missing term in the proportion:

$4 : 6 = 16 : \text{$

$\frac{4}{6} = \frac{16}{x}$ — Cross multiply

$4x = 96$

$\frac{4x}{4} = \frac{96}{4}$ — Simplify: $\div 4$
 $x = 24$

b) Complete the missing term in the proportion:

$5 : \text{$ = $50 : 100$

c) Complete the missing term in the proportion:

20 is to 15 = 8 is to

d) Complete the missing term in the proportion:

$8 : \text{$ = $10 : 15$

e) Find the missing term in the proportion:

$\frac{4}{12} = \frac{y}{9}$
 $4 \cdot 9 = 12 \cdot y$ — Cross multiply

$12y = 36$

$y =$

f) Find the missing term in the proportion:

$\frac{6}{x} = \frac{2}{3}$

$x =$

Skill 9.5 Solving proportions.

- Write the proportion using words.
- Replace the words with numbers:
First the given ratio.
Then the ratio of the given quantity to the unknown quantity (x).
- Rewrite the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find x .

A proportion

$$a : b = c : d \quad \text{2 ratios}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Q. The ratio of concentrated syrup to water is 1 : 4. How much water is needed to dilute 3 pints of concentrated syrup?

A. *syrup : water = syrup : water*
 $1 : 4 = 3 : x$

$$\frac{1}{4} = \frac{3}{x}$$

Cross multiply

$$1 \cdot x = 4 \cdot 3$$

$$x = 12 \text{ pt}$$

a) A risotto recipe uses a ratio of 2 cups of rice to 6 cups of water. How many cups of water have to be added to 6 cups of rice?

$$\text{rice} : \text{water} = \text{rice} : \text{water}$$

$$2 : 6 = 6 : x$$

$$\frac{2}{6} = \frac{6}{x}$$

Cross multiply

$$2x = 36$$

Simplify: $\div 2$ So $x =$

c) The ratio of silver to bronze medals won by Britain at the 2004 Olympics is 3 : 4. If Britain won 9 silver medals, how many bronze medals did it win?

$$=$$

$$=$$

$$\text{So } x =$$

e) The fuel mix for a chainsaw is 4 parts oil to 21 parts gasoline. How much gasoline needs to be added to 8 oz of oil?

$$=$$

$$=$$

$$\text{So } x =$$
 OZ

b) To mix concrete, 2 buckets of sand are added to every 3 buckets of gravel. How much gravel is added to 10 buckets of sand?

$$\text{sand} : \text{gravel} =$$

$$=$$

$$\text{So } x =$$

d) To make a 25% saline solution, 1 part of salt is used for every 3 parts of water. If you use 120 g of salt, how much water do you need to make the saline solution?

$$=$$

$$=$$

$$\text{So } x =$$
 g

f) The Airbus A 380 has a length of 240 ft and a wingspan of 260 ft. A model of this plane has a wingspan of 13 in. How long is the model?

$$=$$

$$=$$

$$\text{So } x =$$
 in.

- Find the total number of equal parts, by adding the numbers in the ratio.
- Calculate what fraction each share represents out of the total number of parts.
- Multiply this fraction by the original quantity.

Q. The ratio of cement to sand to gravel in a concrete mix is 1 : 2 : 3. How much sand is in a 54 pounds concrete mix?

A. $equal\ parts = 1 + 2 + 3 = 6$
 $sand\ share = 2\ out\ of\ 6 = \frac{2}{6} = \frac{1}{3}$
 $sand\ in\ 54\ lb = \frac{1}{3}\ of\ 54\ lb$
 $= \frac{1}{3} \times 54\ lb$ Simplify: $\div 3$
 $= 18\ lb$

a) The 18-carat gold has a ratio of 3 : 1 pure gold to other metals. How many grams of pure gold are needed for a 12 gram necklace?

$equal\ parts = 3 + 1 = 4$
 $pure\ gold\ share = \frac{3}{4}$
 $pure\ gold\ in\ 12\ g = \frac{3}{4}\ of\ 12\ g$
 $= \frac{3}{4} \times 12\ g$ Simplify: $\div 4$ = g

c) The ratio of vowels to consonants in the English language is 5 : 21. How many vowels are likely to be in a 52,000 letter article?

$equal\ parts =$
 $vowels\ share =$
 $vowels\ in\ 52,000\ letters =$
 $=$

e) The ratio of gold to silver to bronze medals won by Canada at the 2004 Olympics is 1 : 2 : 1. If they won 12 medals in total, how many gold medals did Canada win?

$=$

b) The ratio of marriages to divorces in Australia in 2006 was 2 : 1. How many divorces would be likely in 1500 couples?

$equal\ parts =$
 $divorce\ share =$
 $divorces\ in\ 1500\ couples =$
 $=$

d) The fuel mix for a chainsaw is 4 parts oil to 21 parts gasoline. How much gasoline is in a 1500 milliliter chainsaw tank?

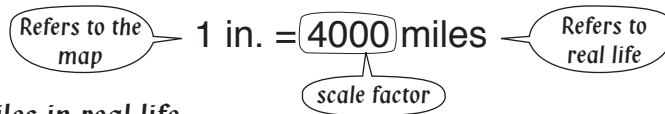
$=$ mL

f) The combined monthly bill for the phone, mobile phone and internet is \$180. If the ratio of phone to mobile phone to internet costs is 5 : 6 : 4, how much is the phone cost?

$=$ \$

Skill 9.7 Working with ratio scales (1).

RATIO SCALE



Hint: 1 inch on the map represents 4000 miles in real life.

Finding the real distance or object dimension (ratio scale and distance on the map or model dimension are given)

- Let x represent the real distance or object dimension.
- Write and solve a proportion:
EITHER
Ratio scale equals the ratio of the map distance and the real distance.
OR
• Ratio scale equals the ratio of the model dimension and the object dimension.

Finding the map distance or model dimension (ratio scale and real distance or object dimension are given)

- Let x represent the map distance or model dimension.
- Write and solve a proportion:
EITHER
Ratio scale equals the ratio of the map distance and the real distance.
OR
• Ratio scale equals the ratio of the model dimension and the object dimension.

Finding the ratio scale (real distance and distance on the map are given)

- Let x represent the scale factor.
- Write and solve a proportion:
EITHER
Ratio scale equals the ratio of the map distance and the real distance.
OR
• Ratio scale equals the ratio of the model dimension and the object dimension.

Q. On a map the scale is 1 in. = 123 miles. What is the distance on the map between Los Angeles and New York, if they are 2460 miles apart?

A. Let x = map distance Write a proportion
 $1 \text{ in.} : 123 \text{ mi} = x \text{ in.} : 2460 \text{ mi}$
 $\frac{1 \text{ in.}}{123 \text{ mi}} = \frac{x \text{ in.}}{2460 \text{ mi}}$ map distance
actual distance
 $\frac{1}{123} \cdot x = \frac{x}{2460}$ Cross multiply
 $123 \cdot x = 1 \cdot 2460$ Simplify
 $123x = 2460$ Divide each side by 123
 $123x \div 123 = 2460 \div 123$ Simplify
 $x = 20$
 The map distance is **20 in.**

a) On a map the scale ratio is 1 cm = 200 km. What is the real distance between Tokyo and Geneva, if they are 80 cm apart on the map?

x = real distance

 $1 \text{ cm} : 200 \text{ km} = 80 \text{ cm} : x \text{ km}$

 $\frac{1}{200} = \frac{80}{x}$ Cross multiply

 $x = 200 \times 80$

 $x =$ km

b) What is the actual distance between Sydney and London, if they are 14 in. apart on a map with a scale ratio of 1 in. = 1500 mi?

x = real distance

 $1 \text{ in.} : 1500 \text{ mi} = 14 \text{ in.} : x \text{ mi}$

 $x =$

 $x =$ mi

Skill 9.7 Working with ratio scales (2).

- c)** The Empire State Building is 1250 ft tall. You are drawing a model using a scale of 1 in. = 125 ft. How high is the model?

$x = \text{model height}$

.....

.....

.....

$x =$

.....

$x =$

in.

- d)** Taipei 101, the world's tallest building, is 500 m high. A model is built using a scale ratio of 1 cm = 10 m. How tall is the model?

$x = \text{model height}$

.....

.....

.....

$x =$

.....

$x =$

cm

- e)** On a map the scale ratio is 1 in. = 250 mi. What is the distance on the map between New York and Munich (Germany), if the real life distance is 4000 miles?

$x = \text{map distance}$

.....

.....

.....

$x =$

.....

$x =$

in.

- f)** On a town plan the scale ratio is 1 in. = 1.5 mi. What is the distance on the map between the airport and the post office, if they are 15 miles apart?

$x = \text{map distance}$

.....

.....

.....

$x =$

.....

$x =$

in.

- g)** Petronas Towers in Kuala Lumpur are 1475 ft tall. What scale is needed to build a model with a height of 25 inches?

$x = \text{scale factor}$

.....

.....

$1 \text{ in.} : x \text{ ft} = 25 \text{ in.} : 1475 \text{ ft}$

.....

$\frac{1}{x} = \frac{25}{1475}$ Cross multiply

.....

$x = 1475 \div 25$

.....

$x =$

1 in. = ft

- h)** The Walt Disney Studios' main theatre screen is 46 ft wide. What scale is needed to build a model with a width of 10 inches?

$x = \text{scale factor}$

.....

.....

.....

$x =$

.....

$x =$

1 in. = ft

$$\text{rate (speed)} = \frac{\text{distance traveled (}d\text{)}}{\text{time taken (}t\text{)}} \quad \text{OR} \quad r = \frac{d}{t}$$

- Write the formula for the rate (speed).
- Convert the given units into the required units if necessary. (see Math Facts, pages 439 and 440)
Hints: If the rate (speed) must be calculated in mph, convert the units for distance to miles and the units for time to hours.
Changing from smaller units into larger units, always divide by the conversion factor.
Changing from larger units into smaller units, always multiply by the conversion factor.
- Substitute the values for distance and time into the formula.
- Evaluate and simplify.

Q. The marine green turtle was recorded swimming 300 miles in 10 days. What was its average speed in mph?

A. $r = \frac{d}{t}$
 where $t = 10 \text{ days} = 10 \times 24 \text{ h} = 240 \text{ h}$

$$r = \frac{300 \text{ mi}}{240 \text{ h}} \quad \text{Substitute into the formula}$$

$$= \frac{\overset{5}{\cancel{300}}}{\underset{4}{\cancel{240}}} \text{ mph} \quad \text{Simplify: } \div 60$$

$$= 1.25 \text{ mph}$$

a) A garden snail named Archie covered a 33 cm course in 2 minutes at the 1995 World Snail Racing Championships, held in England. What was Archie's average speed?

$$t = 2 \text{ min} = \frac{2}{60} \text{ h} = \frac{1}{30} \text{ h} \quad \text{Simplify: } \div 2$$

$$r = \frac{33 \text{ cm}}{\frac{1}{30} \text{ h}} = 33 \div \frac{1}{30} \text{ cm/h}$$

$$= 33 \times \frac{30}{1} \text{ cm/h} = \boxed{} \text{ cm/h}$$

b) The Gentoo penguin in the Antarctic Islands can swim 12.5 miles in half an hour. What is its average speed?

$$t = 30 \text{ min} = \dots\dots\dots$$

$$r = \dots\dots\dots$$

mph

c) The Suzuki Hayabusa is the world's fastest motorbike. It can travel 100 km in 20 minutes. What is its average speed?

$$t = 20 \text{ min} = \dots\dots\dots$$

$$r = \dots\dots\dots$$

km/h

d) Some species of dolphins can swim 4 miles in 10 minutes. What is their average speed in miles per hour?

$$t = \dots\dots\dots$$

$$r = \dots\dots\dots$$

distance traveled (d) = rate (r) \times time taken (t) OR $d = rt$

- Write the formula for distance traveled.
- Convert the given units into the required units if necessary. (see Math Facts, pages 439 and 440)
*Hints: Changing from smaller units into larger units, always divide by the conversion factor.
Changing from larger units into smaller units, always multiply by the conversion factor.*
- Substitute the values for rate and time into the formula.
- Evaluate and simplify.

Q. The ride duration of the Disneyland monorail in California is approximately 15 minutes. What is the length of the ride if the train's average speed is 10 mph?
[Give the answer in miles.]

A. $d = rt$
 where $t = 15 \text{ min} = \frac{15}{60} \text{ h} = \frac{1}{4} \text{ h}$
 $d = 10 \text{ mph} \times \frac{1}{4} \text{ h}$ — *Substitute into the formula*
 $= \cancel{10}^5 \times \frac{1}{\cancel{4}_2} \text{ km}$ — *Simplify: $\div 2$*
 $= 2.5 \text{ mi}$

a) The Russian Alpha class nuclear-powered submarine has a maximum speed of 46.6 mph, or 40 knots. At this speed, what distance can it cover in 12 hours?

Use $d = rt$

$d = 46.6 \text{ mph} \times 12 \text{ h}$
 $= 46.6 \times 12 \text{ mi} = \boxed{559.2 \text{ mi}}$

b) The Japanese Bullet train is the fastest scheduled train service in the world with an average speed of 260 km/h. At this speed, what distance can it cover in 3 hours?

$d =$
 $=$ $\boxed{\text{ km}}$

c) The Nile flows at an average speed of around 7.5 km/h during inundation season. At this speed, what distance might a boat floating on the Nile travel in 6 hours?

$d =$
 $=$ $\boxed{\text{ km}}$

d) The Eurostar trains operate from London to Brussels and run at an average speed of 87 mph. If the trip takes two and half hours, what is the distance from London to Brussels?

$d =$
 $=$ $\boxed{\text{ mi}}$

e) A garden snail can travel at 0.012 m/s. At this speed, what distance can it cover in 10 minutes?

Use $d = rt$

$t = 10 \text{ min} = 10 \times 60 \text{ s} = 600 \text{ s}$
 $d =$ $\boxed{\text{ m}}$

f) The F-16 Falcon fighter aircraft can fly at a speed of 1500 miles per hour at sea level. At this speed, what distance can it cover in 20 min?

$t =$
 $d =$ $\boxed{\text{ mi}}$

$$\text{time taken } (t) = \frac{\text{distance traveled } (d)}{\text{rate } (r)} \quad \text{OR} \quad t = \frac{d}{r}$$

- Write the formula for the time taken.
- Convert the given units into the required units if necessary. (see Math Facts, pages 439 and 440)
*Hints: Changing from smaller units into larger units, always divide by the conversion factor.
 Changing from larger units into smaller units, always multiply by the conversion factor.*
- Substitute the values for the distance traveled and the rate into the formula.
- Evaluate and simplify.

Q. Donghai Bridge China is the longest cross-sea bridge in the world, with a length of 32 km. How long will it take a car travelling at 80 km/h to cross the bridge? [Give the answer in hours.]

A. $t = \frac{d}{r}$
 $= \frac{32 \text{ km}}{80 \text{ km/h}}$ *Substitute into the formula*
 $= \frac{2 \cancel{32}}{5 \cancel{80}} \text{ h}$ *Simplify: $\div 16$*
 $= 2 \div 5 \text{ h}$
 $= 0.4 \text{ h}$

a) The speed of long distance jogging for an average person is around 6.5 mph. At this speed, how long will it take a person to run 13 miles?

$$t = \frac{13 \text{ mi}}{6.5 \text{ mph}}$$

$$= 13 \div 6.5 \text{ h} = \boxed{} \text{ h}$$

b) Michael Milton is the world's fastest skier on one leg with a record of nearly 60 m/s. At this rate, how long will it take him to ski 3000 m?

$$t =$$

$$= = \boxed{} \text{ s}$$

c) Top athletes can sprint at a speed around 10 m/s within a short distance. How long will it take an athlete to sprint 200 m?

$$t =$$

$$= = \boxed{} \text{ s}$$

d) The average speed of a space shuttle in orbit is 5000 miles/s. At this speed, how long will it take a space shuttle to travel 1000 miles?

$$t =$$

$$= = \boxed{} \text{ s}$$

e) The average walking speed for female adults is 3 mph. At this speed, how long will it take a female adult to walk 5 miles?

$$t =$$

$$= = \boxed{} \text{ min}$$

f) The Metro monorail in Sydney is 3.6 km long. At an average speed of 24 km/h, how long would it take the train to complete a loop?

$$t =$$

$$= = \boxed{} \text{ min}$$

$$\text{rate} = \frac{\text{amount}}{\text{time}}$$

Rate of change
(amount and time are given)

- Convert the given units to the required units. (see Math Facts, pages 439 and 440)
- Divide the amount by the time taken.

Example: A 300 L bathtub can be filled in 10 minutes.

$$\text{Rate} = \frac{300 \text{ L}}{10 \text{ min}} = 30 \text{ L/min}$$

$$\text{amount} = \text{rate} \times \text{time}$$

Amount
(rate and time are given)

- Convert the given units to the required units. (see Math Facts, pages 439 and 440)
- Multiply the rate by the time taken.

Example: Sam worked 7 hours and was paid at a rate of \$16/h.

$$\text{Amount (pay)} = 16 \times 7 = \$112$$

$$\text{time} = \frac{\text{amount}}{\text{rate}}$$

Time taken
(amount and rate are given)

- Convert the given units to the required units. (see Math Facts, pages 439 and 440)
- Divide the amount by the rate.

Example: A Lexmark E232 prints 990 pages at a rate of 22 pages/min (ppm).

$$\text{Time} = \frac{990 \text{ p}}{22 \text{ ppm}} = 45 \text{ min}$$

Q. The average pulse for a new born baby is around 130 beats per minute. How many beats in 3 hours is this?

A. $\text{amount} = \text{rate} \times \text{time}$

$$\text{rate} = 130 \text{ beats/min}$$

$$\text{time} = 3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$$

$$\begin{aligned} \text{amount} &= 130 \text{ beats/min} \times 180 \text{ min} \\ &= \mathbf{23,400 \text{ beats}} \end{aligned}$$

a) The ruby throated hummingbird can beat its wings around 21,000 times in 5 minutes. What is its wing beat rate in beats per second?

$$\text{time} = 5 \times 60 \text{ s} = 300 \text{ s}$$

$$\text{rate} = \frac{21,000 \text{ beats}}{300 \text{ s}} = \boxed{\text{beats/s}}$$

Simplify: $\div 100$ then $\div 3$

b) The heart of an unborn baby beats at around 9000 times in an hour. What is the heart rate of an unborn baby in beats per minute?

$$\text{time} = 1 \times 60 \text{ min} = 60 \text{ min}$$

$$\text{rate} = \frac{\text{amount}}{\text{time}} = \boxed{\text{beats/min}}$$

c) Find the time taken to print 875 sheets of paper, if a Lexmark T640 printer can print 35 pages per minute.

$$\text{time} = \frac{\text{amount}}{\text{rate}}$$

$$= \frac{875}{35} = \boxed{\text{min}}$$

d) An adult's air intake adds up to 7500 L per day. How many liters of air does an adult breathe in a week?

$$\text{amount} = \text{rate} \times \text{time}$$

$$= 7500 \text{ L/day} \times 7 \text{ days} = \boxed{\text{L}}$$

e) In 2006 Australia was the world's second least densely populated country with 2.7 people/km². If Australia has an area of nearly 7.7 million km², what was its population in 2006?

$$\text{population} = \text{area} \times \text{density rate}$$

$$= 7.7 \text{ million km}^2 \times 2.7 \text{ people/km}^2 = \boxed{\text{population}}$$

f) Bangladesh is the most densely populated country, with around 1050 people per km² in 2007. If in 2007 Bangladesh had a population of 151,200,000, what is its area in km²?

$$\text{area} = \text{population} \div \text{density rate}$$

$$= \frac{151,200,000}{1050 \text{ people/km}^2} = \boxed{\text{area}}$$

- Write the rates as fractions.
- Bring the rates to the same unit. (see Math Facts, pages 439 and 440)
Hint: It's easier to change from larger units into smaller units, because you multiply by the conversion factor.
- Evaluate and simplify.
- Compare the numbers.

Q. Which density is lower?

- A) 670 kg/m³ (gasoline)
B) 1.025 g/cm³ (seawater)

A. kg/m³ to g/cm³: kg to g ⇒ × 1000

$$m^3 \text{ to } cm^3 \Rightarrow \times 1,000,000$$

$$A) \text{ density} = \frac{670 \text{ kg}}{1 \text{ m}^3}$$

$$= \frac{670 \times 1000 \text{ g}}{1 \times 1,000,000 \text{ cm}^3}$$

cross off respective 0's

$$= \frac{67}{100} \text{ g/cm}^3$$

$$= 0.67 \text{ g/cm}^3$$

$$B) \text{ density} = 1.025 \text{ g/cm}^3$$

0.67 < 1.025, so the answer is **A)**

a) Which heartbeat rate is higher?

- A) 70 beats/min (adult)
B) 2 beats/s (baby)

$$B) \text{ rate} = \frac{2 \text{ beats}}{1 \text{ s}} = \frac{2 \text{ beats}}{\frac{1}{60} \text{ min}}$$

$$= 2 \div \frac{1}{60} = 2 \times \frac{60}{1} = 120 \text{ beats/min}$$

$$120 \text{ beats/min} > 70 \text{ beats/min}$$

B

b) Which heartbeat rate is lower?

- A) 1.5 beats/s (child)
B) 70 beats/min (adult)

$$A) \text{ rate} =$$

c) Which birth rate is lower?

- A) 12.1 per 1000 people (Australia)
B) 82 per 10,000 people (Germany)

$$A) \text{ rate} = \frac{12.1}{1000} =$$

d) Which birth rate is higher?

- A) 14.1 per 1000 people (USA)
B) 94 per 10,000 people (Japan)

e) Which density is lower?

- A) 800 kg/m³ (petrol)
B) 1 g/cm³ (water)

f) Which density is higher?

- A) 2.2 g/cm³ (graphite)
B) 1300 kg/m³ (PVC)