

12. [Operations]

Skill 12.1 Using the commutative property for addition.

MMYellow 1 1 2 2 3 3 4 4
MMRed 1 1 2 2 3 3 4 4

Example:

$$2 + 5 = 7$$

$$5 + 2 = 7$$

COMMUTATIVE LAW for ADDITION
You can add numbers in any order
and not change the outcome.

SO $2 + 5 = 5 + 2$

Q. True or false?

$$6 + 3 = 3 + 6$$

A. *true*

Solve both sides of the equation and compare the results.

$$6 + 3 = 9$$

$$3 + 6 = 9$$

The results are the same.

a) True or false?

$$10 - 4 = 4 - 10$$

$10 - 4 = 6$ but $4 - 10 \neq 6$

b) True or false?

$$4 + 5 = 5 + 4$$

c) True or false?

$$7 + 9 = 9 + 7$$

d) True or false?

$$9 - 3 = 3 - 9$$

e) + 2 = 2 + 8

f) $9 + 6 =$ + 9

g) $4 + 1 =$ + 4

h) + 5 = 5 + 2

i) + 7 = 7 + 5

j) $3 + 9 =$ + 3

k) + 13 = 13 + 6

l) $17 + 10 =$ + 17

m) $11 + 19 =$ + 11

n) + 22 = 22 + 14

o) + 17 = 17 + 12

p) $15 + 18 =$ + 15

Example:

$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

COMMUTATIVE LAW for MULTIPLICATION

You can multiply numbers in any order and not change the outcome.

SO $2 \times 5 = 5 \times 2$

Q. $\times 5 = 5 \times 9$

A. 9

Ask: "What number multiplied by 5 equals 5 multiplied by 9?"

Answer: $9 \times 5 = 5 \times 9$

a) True or false?

$$10 \div 2 = 2 \div 10$$

$10 \div 2 = 5$ but $2 \div 10 \neq 5$ false

b) True or false?

$$4 \times 5 = 5 \times 4$$

.....

c) True or false?

$$7 \times 9 = 9 \times 7$$

.....

d) True or false?

$$9 \div 3 = 3 \div 9$$

.....

e) 8 $\times 2 = 2 \times 8$

f) $\times 5 = 5 \times 2$

g) $4 \times 1 =$ $\times 4$

h) $9 \times 6 =$ $\times 9$

i) $\times 4 = 4 \times 12$

j) $19 \times 10 =$ $\times 19$

k) $11 \times 3 =$ $\times 11$

l) $\times 6 = 6 \times 18$

m) $\times 13 = 13 \times 12$

n) $7 \times 18 =$ $\times 7$

o) $\times 24 = 24 \times 17$

p) $13 \times 15 =$ $\times 13$

Example:

$$14 + 0 = 14$$

IDENTITY LAW of ADDITION using ZERO
The sum of zero and any number is that number.

Q. + 0 = 2

A. 2

Ask: "What number added to zero makes 2?"

Answer: $2 + 0 = 2$

a) True or false?
 $10 + 0 = 10$

b) True or false?
 $6 + 0 = 0$

c) True or false?
 $0 + 7 = 7$

d) True or false?
 $0 + 8 = 8$

e) True or false?
 $3 - 0 = 0$

f) True or false?
 $9 - 0 = 9$

g) + 0 = 8

h) + 0 = 5

i) $3 + 0 =$

j) $9 +$ $= 9$

k) $2 -$ $= 0$

l) $5 -$ $= 5$

m) Which expression equals 7?

- A) $0 + 7$
- B) 0×7
- C) $0 - 7$

n) Which expression equals 8?

- A) 0×8
- B) $0 - 8$
- C) $0 + 8$

o) Which expression equals 6?

- A) 6×0
- B) $0 - 6$
- C) $6 + 0$

p) Which expression equals 3?

- A) $3 + 0$
- B) $0 - 3$
- C) 3×0

Example:

$$14 \times 1 = 14$$

IDENTITY LAW for MULTIPLICATION using ONE
The product of one and any number is that number.

Q. $\times 1 = 8$ **A.** **8** Ask: “What number multiplied by 1 makes 8?”
Answer: $8 \times 1 = 8$

Q. Which expression equals 13? **A.** **B** Solve all expressions and then compare the results.
A) $1 + 13$ A) $1 + 13 \neq 13$
B) 1×13 B) $1 \times 13 = 13$
C) $1 \div 13$ C) $1 \div 13 \neq 13$

a) True or false?
 $6 \times 1 = 6$

b) True or false?
 $1 \times 4 = 4$

c) True or false?
 $1 \times 1 = 2$

d) True or false?
 $9 \times 1 = 9$

e) $\times 1 = 2$

f) $\times 1 = 7$

g) $3 \times 1 =$

h) $5 \times$ $= 5$

i) Which expression equals 4?
A) 1×4
B) $1 + 4$
C) $1 \div 4$

j) Which expression equals 12?
A) $1 + 12$
B) $1 \div 12$
C) 1×12

Example:

$$(2 + 5) + 5 = (7) + 5 = 12$$

$$2 + (5 + 5) = 2 + (10) = 12$$

ASSOCIATIVE LAW for ADDITION
Rearranging the grouping of numbers does not change their sum.

SO $(2 + 5) + 5 = 2 + (5 + 5)$

Q. Regroup the expression by writing an easier sum that uses a multiple of 10.
 $(16 + 7) + 3$

A. $16 + (7 + 3)$
Find which two numbers add to 10.
Group them with brackets.
It is easier to add to a 10!

a) Which expression equals $3 + (7 + 9)$?
A) $3 - (7 + 9)$
B) $(3 + 7) + 9$
C) $3 + 7 - 9$

b) Which expression does not equal $2 + 8 + 5$?
A) $2 + (8 + 5)$
B) $(2 + 8) + 5$
C) $2 + 8 - 5$

c) Regroup the expression by writing an easier sum that uses a multiple of 10.
 $5 + (5 + 19)$

d) Regroup the expression by writing an easier sum that uses a multiple of 10.
 $(5 + 17) + 3$

e) Regroup the expression by writing an easier sum that uses a multiple of 10.
 $(9 + 8) + 2$

f) Regroup the expression by writing an easier sum that uses a multiple of 10.
 $4 + (6 + 17)$

g) Simplify by adding the numbers in a different order:
 $14 + (6 + 18) =$
 $(14 + 6) + 18 = 20 + 18 =$

h) Simplify by adding the numbers in a different order:
 $(9 + 18) + 12 =$
.....

i) Simplify by adding the numbers in a different order:
 $(8 + 13) + 7 =$
.....

j) Simplify by adding the numbers in a different order:
 $19 + (11 + 27) =$
.....

Example:

$$(2 \times 5) \times 5 = (10) \times 5 = 50$$

$$2 \times (5 \times 5) = 2 \times (25) = 50$$

ASSOCIATIVE LAW for MULTIPLICATION

Rearranging the grouping of numbers does not change their product.

SO $(2 \times 5) \times 5 = 2 \times (5 \times 5)$

- Q.** Regroup the expression by writing an easier sum that uses a multiple of 10.

$$(9 \times 5) \times 4$$

A. $9 \times (5 \times 4)$

Find which two numbers multiplied result in a multiple of 10.

$$9 \times 5 = 45$$

$$5 \times 4 = 20$$

Group 5 and 4 with brackets.

- a)** Which expression equals

$$3 \times (7 \times 7)?$$

A) $7 \div (7 \times 3)$

B) $(3 \times 7) \times 7$

C) $3 \times 7 \div 7$

- b)** Which expression does not equal

$$2 \times 8 \times 4?$$

A) $2 \times (8 \times 4)$

B) $(2 \times 8) \times 4$

C) $2 \times 8 \div 4$

- c)** Regroup the expression by writing an easier sum that uses a multiple of 10.

$$5 \times (2 \times 37)$$

- d)** Regroup the expression by writing an easier sum that uses a multiple of 10.

$$(14 \times 4) \times 5$$

- e)** Regroup the expression by writing an easier sum that uses a multiple of 10.

$$4 \times (25 \times 8)$$

- f)** Regroup the expression by writing an easier sum that uses a multiple of 10.

$$(17 \times 4) \times 25$$

- g)** Simplify by multiplying the numbers in a different order:

$$(9 \times 25) \times 4 =$$

$$9 \times (25 \times 4) = 9 \times 100 =$$

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- h)** Simplify by multiplying the numbers in a different order:

$$(13 \times 5) \times 2 =$$

.....

- i)** Simplify by multiplying the numbers in a different order:

$$4 \times (25 \times 11) =$$

.....

- j)** Simplify by multiplying the numbers in a different order:

$$(8 \times 2) \times 25 =$$

.....

The inverse (opposite) operation of addition is subtraction.

When you perform two inverse operations (adding 5 and subtracting 5) on a number (17) the number stays unchanged.

Example:

$$\begin{aligned} & 17 + 5 - 5 \\ & = 22 - 5 \\ & = 17 \end{aligned}$$

INVERSE OPERATIONS + AND -
Subtraction of a number undoes addition of that same number.

The inverse (opposite) operation of multiplication is division.

When you perform two inverse operations (multiplying by 5 and dividing by 5) on a number (7) the number stays unchanged.

Example:

$$\begin{aligned} & 7 \times 5 \div 5 \\ & = 35 \div 5 \\ & = 7 \end{aligned}$$

INVERSE OPERATIONS × AND ÷
Division by a number undoes multiplication by that same number.

Q. $24 + \boxed{} - 7 = 24$

A. **7**

Check

$$\begin{aligned} & 24 + ? - 7 = 24 \\ \Rightarrow & 24 - 7 = 17 \\ \Rightarrow & 17 + ? = 24 \\ \Rightarrow & 17 + \mathbf{7} = 24 \end{aligned}$$

Ask: "What number, added to 24 undoes subtracting 7?"

Answer: Adding 7 undoes subtracting 7.

Check the result.

a) $14 \times 2 \div \boxed{2} = 14$

b) $20 + 12 - 12 = \boxed{}$

$28 \div ? = 14$

c) $18 + 3 \boxed{} 3 = 18$

d) $25 \div 5 \boxed{} 5 = 25$

e) $18 \boxed{} 3 - 3 = 18$

f) $16 \times \boxed{} \div 4 = 16$

g) $9 \times 7 \div \boxed{} = 9$

h) $32 \div 8 \boxed{} 8 = 32$

- Add (+) and/or subtract (-) from left to right.

Q. $8 - 2 - 5 + 6 =$

A. $8 - 2 - 5 + 6 =$
 $= 6 - 5 + 6$
 $= 1 + 6$
 $= 7$

Start with 8 and subtract 2.
 The result is 6.
 Then subtract 5 from 6.
 The result is 1.
 Finally add 6 to the 1.

a) $8 + 2 + 4 =$

$10 + 4 =$

14

b) $6 + 5 - 3 =$

.....

c) $14 - 7 - 6 =$

.....

d) $7 - 5 + 9 =$

.....

e) $19 - 8 + 1 =$

.....

f) $16 - 2 + 5 =$

.....

g) $4 + 6 + 3 =$

.....

h) $13 - 7 - 4 =$

.....

i) $5 + 8 - 9 =$

.....

j) $6 + 5 + 1 - 2 =$

$11 + 1 - 2 =$

$12 - 2 =$

10

k) $8 - 4 + 3 + 2 =$

.....

l) $9 + 7 - 5 - 1 =$

.....

m) $7 + 3 + 5 - 6 =$

.....

n) $5 - 2 + 7 - 5 =$

.....

o) $9 - 3 - 2 - 1 =$

.....

p) $5 + 8 - 4 - 3 =$

.....

q) $29 - 4 + 7 + 2 =$

.....

r) $8 + 16 - 9 + 5 =$

.....

- Multiply (\times) and/or divide (\div) from left to right.

Q. $12 \div 3 \times 5 =$

A. $12 \div 3 \times 5 =$
 $= 4 \times 5$
 $= 20$

Start with 12 and divide by 3.
The result is 4.
Then multiply 4 by 5.

a) $2 \times 5 \times 3 =$

$10 \times 3 =$

b) $5 \times 3 \div 3 =$

.....

c) $16 \div 4 \div 2 =$

.....

d) $12 \div 3 \times 4 =$

.....

e) $2 \times 1 \times 3 =$

.....

f) $14 \div 7 \times 4 =$

.....

g) $5 \times 4 \div 4 =$

.....

h) $18 \div 6 \div 3 =$

.....

i) $7 \times 2 \div 7 =$

.....

j) $4 \times 2 \times 2 =$

$8 \times 2 =$

k) $2 \times 9 \div 6 =$

.....

l) $20 \div 5 \div 2 =$

.....

m) $35 \div 5 \times 6 =$

.....

n) $3 \times 4 \times 5 =$

.....

o) $24 \div 4 \times 2 =$

.....

p) $3 \times 4 \div 3 =$

.....

q) $56 \div 7 \div 2 =$

.....

r) $6 \times 6 \div 3 =$

.....

- Multiply (×) and/or divide (÷) from left to right.
- Add (+) and/or subtract (-) from left to right.

Q. $6 + 12 \div 3 =$

A. $6 + 12 \div 3 =$
 $= 6 + 4$
 $= 10$

First do 12 divided by 3.
 The result is 4.
 Then add 6 and 4.

a) $21 \div 3 - 2 =$

$7 - 2 =$

5

b) $4 + 3 \times 3 =$

.....

c) $6 \times 2 + 8 =$

.....

d) $15 \div 5 - 2 =$

.....

e) $2 \times 5 - 4 =$

.....

f) $6 + 3 \times 5 =$

.....

g) $6 + 9 \div 3 =$

.....

h) $18 \div 2 + 4 =$

.....

i) $3 \times 4 + 7 =$

.....

j) $13 - 3 \times 3 =$

$13 - 9 =$

4

k) $4 \times 4 - 7 =$

.....

l) $15 - 10 \div 5 =$

.....

m) $56 \div 7 - 1 \times 4 =$

.....

n) $8 + 12 \div 4 - 2 =$

.....

o) $45 - 5 \times 2 - 5 =$

.....

p) $38 - 12 \div 2 \times 3 =$

.....

q) $16 \div 4 \times 8 + 4 =$

.....

r) $18 \div 6 \times 9 - 4 =$

.....

- Simplify within the brackets.
- Multiply (\times) and/or divide (\div) from left to right.
- Add (+) and/or subtract ($-$) from left to right.

Q. $9 + 12 \div (9 - 5) =$

A. $9 + 12 \div (9 - 5) =$
 $= 9 + 12 \div 4$
 $= 9 + 3$
 $= \mathbf{12}$

Simplify inside the brackets and subtract 5 from 9. The result is 4. Then divide 12 by 4. The result is 3. Finally add 9 and 3.

a) $7 \times (4 - 2) =$

$7 \times 2 =$

14

b) $9 - (4 + 3) =$

c) $(5 - 2) + 7 =$

d) $(8 + 3) \times 2 =$

e) $(4 + 4) \times 3 =$

f) $15 \div (5 - 2) =$

g) $(17 - 8) \times 2 =$

h) $18 \div (6 + 3) =$

i) $(28 - 20) \times 7 =$

j) $8 + (5 + 1) \div 2 =$

$8 + 6 \div 2 =$

$8 + 3 =$

11

k) $4 + (2 + 3) \times 2 =$

l) $15 \div 3 - (2 + 2) =$

m) $9 + (7 - 4) \times 3 =$

n) $18 \div (9 - 3) + 2 =$

o) $9 + 3 \times (8 - 4) =$